Studies of ”electron heating” using MAD-X

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Motivation and introduction

- Besides cooling, electron cooler acts as nonlinear optical element leading to excitation of resonances, and so to possibly resulting in emittance growth (effective "heating") → CELSIUS (V. Ziemann, TSL-note 98-43)
- Within FAIR, beams with high requirements to beam quality → electron cooling necessary especially for secondary beams in storage rings
- SchwerIonenSynchrotron (SIS) - 18:
  - has electron cooler
  - high currents leading to large space charge tune shift and spread → high probability that tune footprint crosses resonance
  - possibly experiment on this topic
- Extension of beambeam element by additional radial density profiles to describe interaction between particle beam and electron beam in MAD-X

Linear tune shift

- First rough estimate for influence of electron cooler from according linear tune shift
- Tune shift for electron beam with rectangular density may be written as

$$|\Delta\nu_z| = \frac{e}{8\pi^2\epsilon_0 m_0 c^2}\frac{\hat{\beta}_z e N'}{\beta^2\gamma^3 b^2},$$

where $\hat{\beta}_z$ – beta function with $z = x, y$

N' – electron number in the cooler

β, γ – relativistic factors;

b – electron beam radius

- Stronger influence for small energies $\longrightarrow$ visible effect in SIS - 18 expected
- Includes electrostatic and magnetic interaction (2 parallel currents)

Comparison of linear tune shifts

	SIS - 18[1]	HESR[2]
pc/GeV	0.15 (injection)	1.5 (minimum)
γ	1.01	1.89
β	0.15	0.85
I_e/A	0.3	0.2
b/mm	22	5
$l_{\mathrm{cooler}}/\mathrm{m}$	3	25
$\hat{\beta}_x/\mathrm{m}$	8	20
$\lvert\Delta\nu_x\rvert$	0.033	0.005

[1] B. Franczak, SIS Parameter List; L. Groening, GSI-dissertation 98-20.

[2] HESR Technical baseline report.

Analytic model for Validation

- Model taken from V. Ziemann, TSL-note 2004-60, is extension of model from E. Keil (CERN)
- Aim: Calculation of nonlinear tune shifts and footprints with electron beams with different density profiles and represented by a single kick
- using integral expressions for those:

$$\Delta\nu_x(J_x, J_y) = 2\int_0^{2\pi}\frac{d\phi_x}{2\pi}\int_0^{2\pi}\frac{d\phi_y}{2\pi}\cos^2\phi_x\frac{\tilde{S}(\sqrt{2J_x\cos^2\phi_x + 2J_y\cos^2\phi_y})}{\sqrt{2J_x\cos^2\phi_x + 2J_y\cos^2\phi_y}}$$

with action and angle variables, $J_{x/y} = \epsilon_{x/y}/2$ and $\phi_{x/y}$, of betatron motion

i.e. $x = \sqrt{2J_x\beta_x}\cos\phi_x, \;\; y$ respectively,

and the normalized electrostatic force function $\tilde{S}$

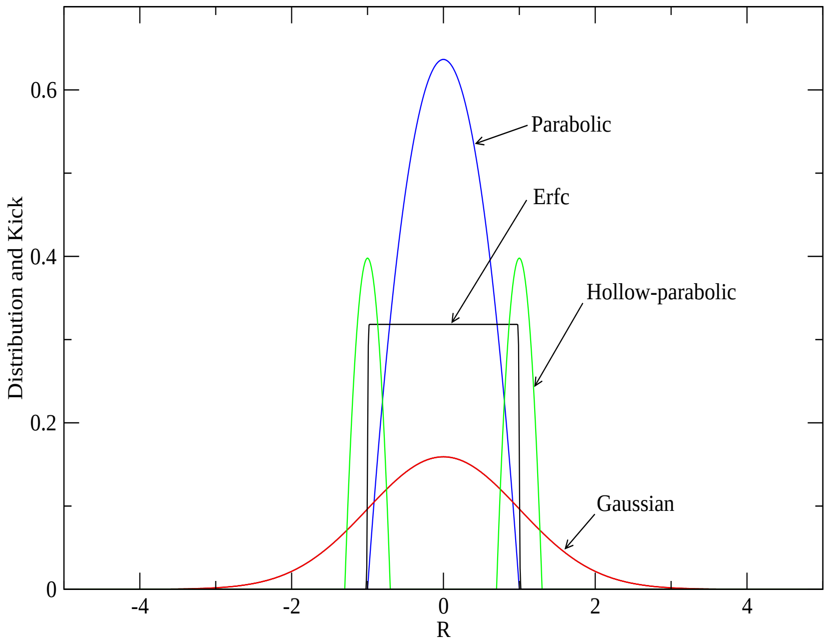
Analytic model for Validation

- Resonance strength from Fourier representation

$$\sqrt{2J}\cos\phi\,\tilde{S}(\sqrt{2J}\cos\phi) = J\sum_{n=0}^{\infty} A_n \cos n\phi$$

- Cooler kicks with several radial density distributions: Gaussian, Parabolic, Error function (Flat top), and Hollow parabolic
- beambeam element with these shapes in MAD-X implemented excepted for parabolic profile

Analytic model: electron density profiles used



Charge normalized to 1, i.e.

$$2\pi \int_0^\infty \mathrm{d}r \; r \; n_0(r) = 1$$

- **Figure from V. Ziemann, TSL-note 2004-60, density profiles used: Gaussian, flat top (error function), hollow-parabolic, and parabolic**

Model for tracking

- Model system for tracking: rotational matrix + single beambeam kick

$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_{0,x} & \hat{\beta}_{0,x} \sin 2\pi\nu_{0,x} \\ -\frac{1}{\hat{\beta}_{0,x}} \sin 2\pi\nu_{0,x} & \cos 2\pi\nu_{0,x} \end{pmatrix} \begin{pmatrix} x_n \\ x'_n + \Delta x'(x_n, y_n) \end{pmatrix}$$

with:

– phase space variables in ith step x_i, x'_i

– beta function in the cooler $\hat{\beta}_0$

– phase advance in the lattice $2\pi\nu_0$

– transverse momentum kick due to electron beam $\Delta x'(x_n, y_n)$ (thin lens approximation)

Transverse momentum kick

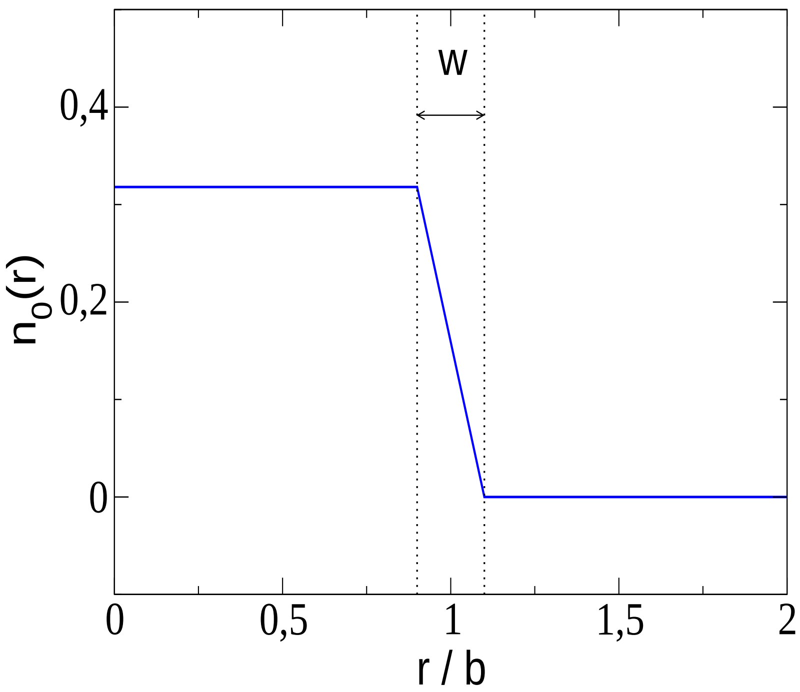
$$\Delta x'(x,y) = \frac{qq'N'}{2\pi\varepsilon_0 m_0 c^2\beta^2\gamma^3}\frac{x}{R^2}\int_0^R \mathrm{d}r\; r\; n_0(r)$$

with

- q, q' – particle charge in considered beam (protons),
  "colliding" beam (electrons)
- $N' = I_e L_{\mathrm{cool}}/(\beta c)$ – number of electrons in the cooler
- $R = \sqrt{x^2+y^2}$ – distance from the centre of the electron beam
- radial electron current density is normalized by (round electron beam)

$$\int_0^\infty \mathrm{d}r\; r\; n_0(r) = 1$$

Electron beam profiles in MAD-X



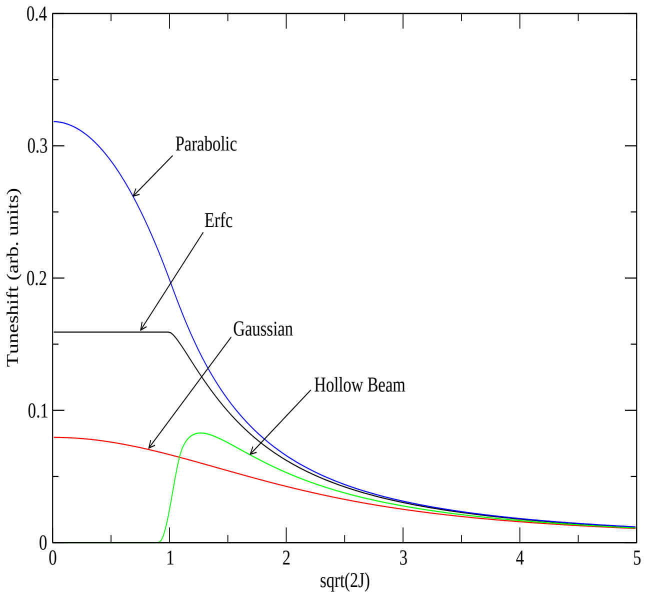
- **beambeam kick elements in MAD-X with**
  - **Gaussian profile (initial)**
  - **Flat top profile (new)**
  - **Hollow-parabolic profile (new)**
- **Flat top profile for electron cooler important**
- **Here, trapezoidal profile used with beam radius b and width of edge layer w**
- **Advantage: analytical expressions for resulting force and maps**

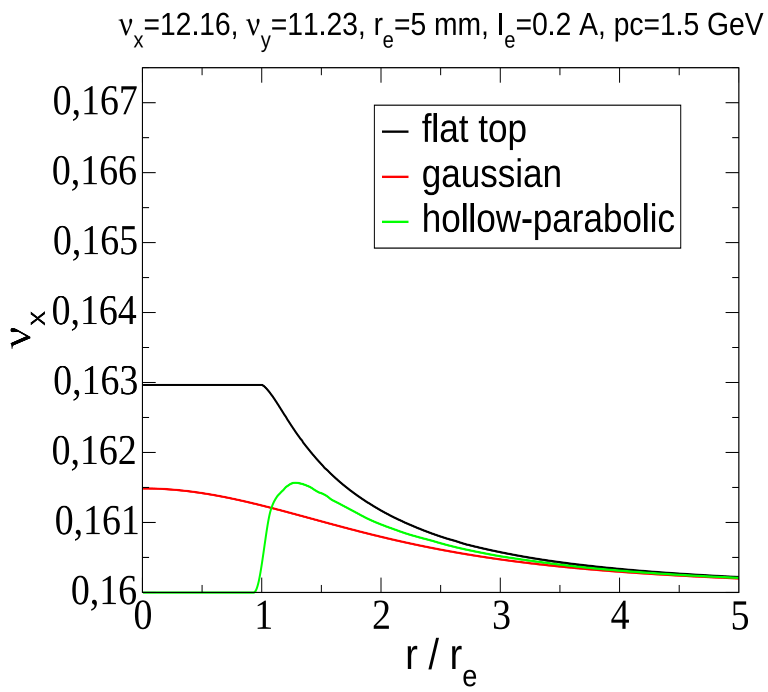
MAD-X with kick vs. analytic model

Non-linear tune shift due to the kick: **protons** $\Rightarrow$ tune shift is positive

left: analytic, **right:** MAD-X, rotational matrix + single kick

HESR conditions: $(\nu_x = 12.16,\ \nu_y = 11.23)$[1], $I_e = 0.2$ A, $pc = 1.5$ GeV





$$\frac{\hat{\beta}_z e^2 N'}{4\pi\varepsilon_0 m_0 c^2 \beta^2 \gamma^3} \frac{1}{2\pi b^2}$$

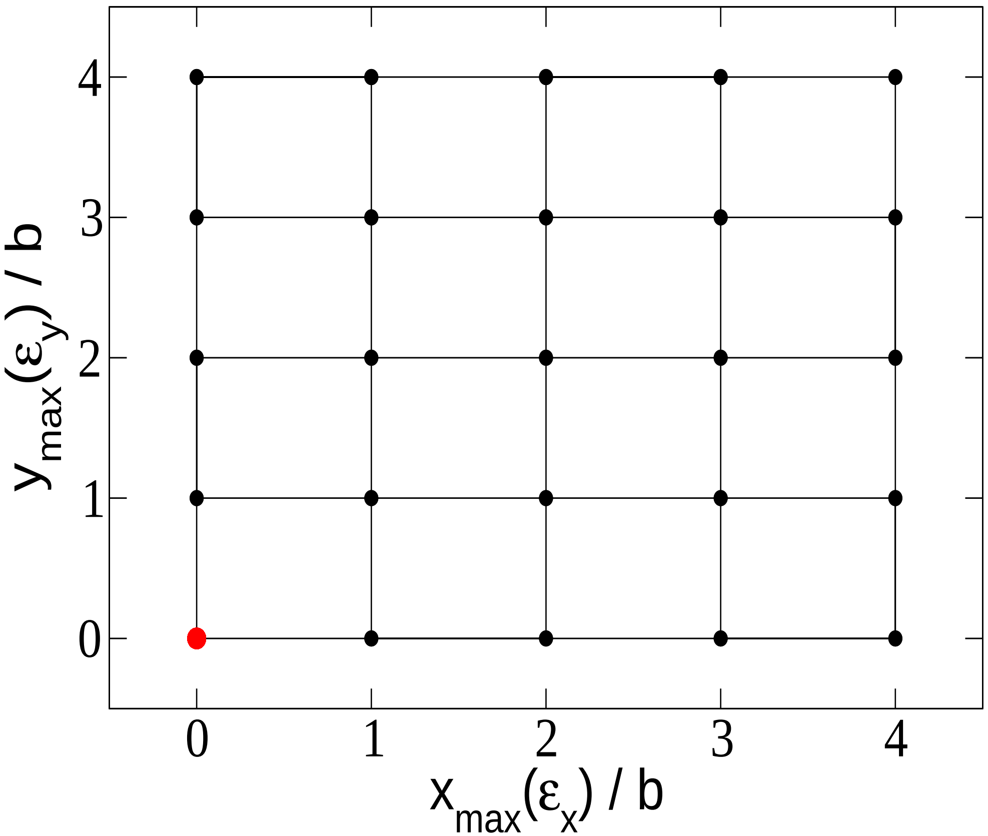
$$= \frac{1}{2\pi} \text{ (left)}$$

$$= 2.98 \cdot 10^{-3} \text{ (right)}$$

[1] Yu. Senichev, "The advanced HESR lattice for the stochastic cooling", FZ Jülich 2006

MAD-X with kick vs. analytic model

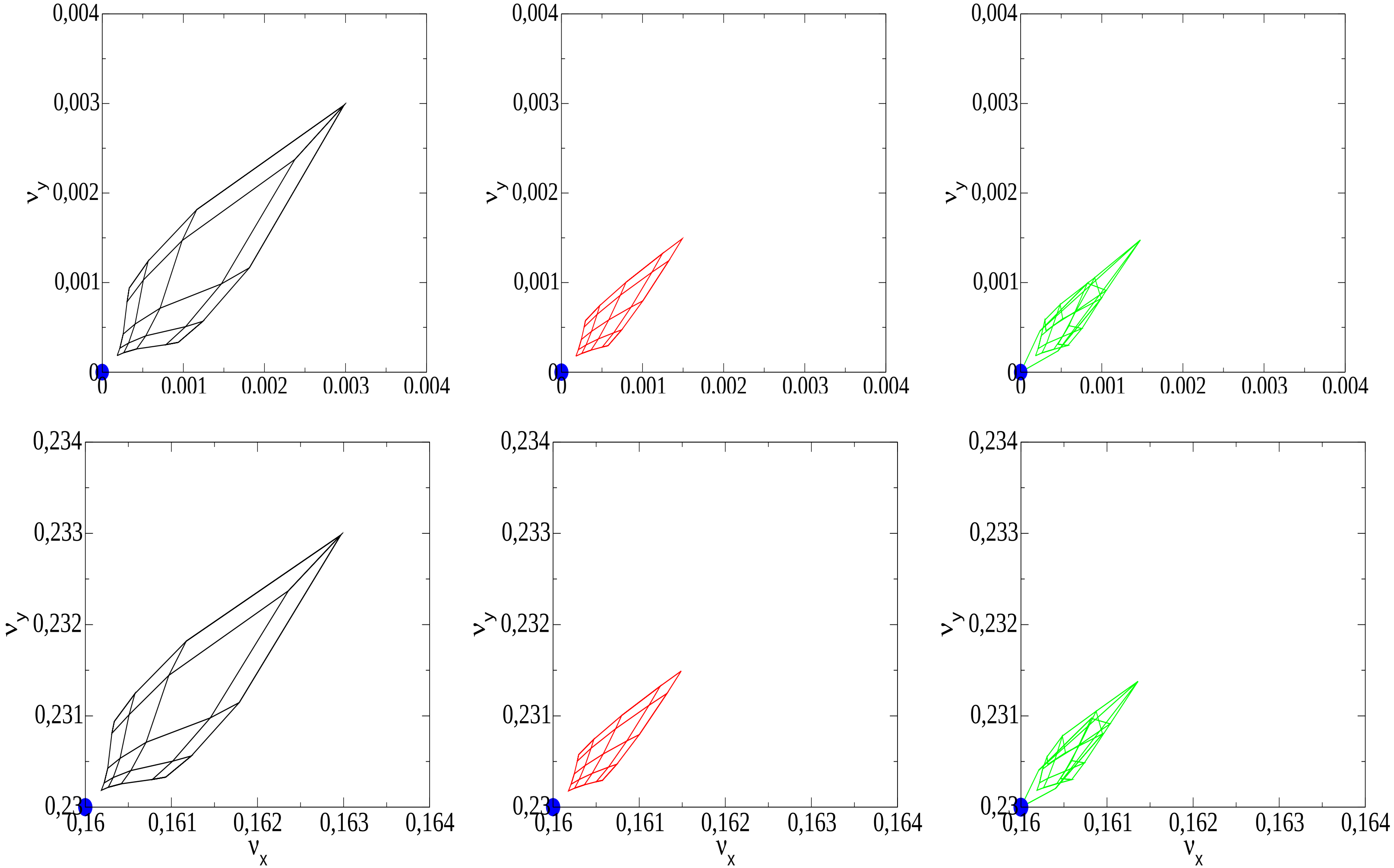
Tune footprint



- **Initial distribution chosen to make shape and structure of tune spreads better visible**
- $z_{\max} = \sqrt{\varepsilon_z \hat{\beta}_z}, \ z = x, y$

MAD-X with kick vs. analytic model

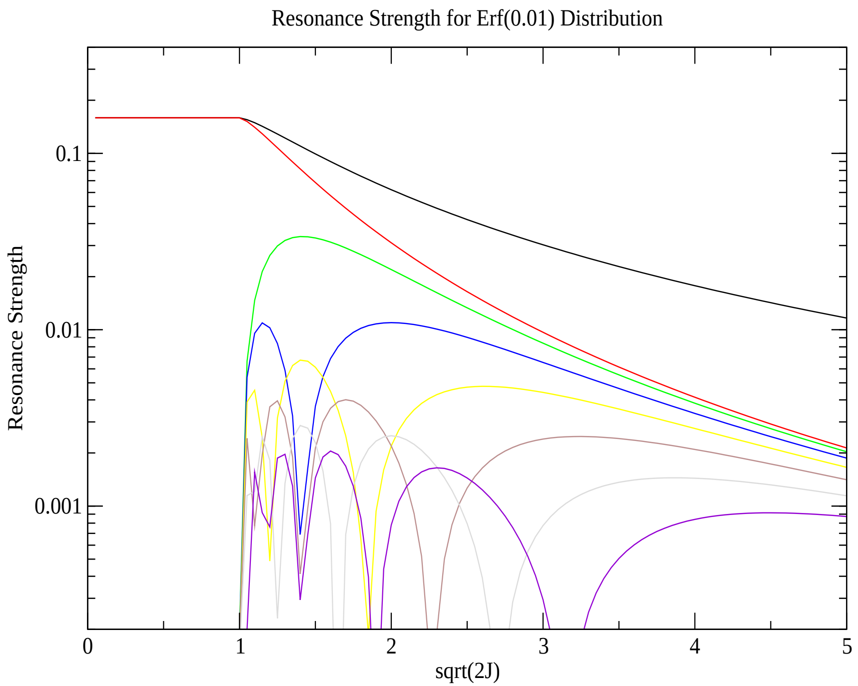
Tune footprints up: analytic, down: MAD-X, rotational matrix + single kick



Protons with $\nu_x = 12.16, \nu_y = 11.23, r_e = 5 \text{ mm}, I_e = 0.2 \text{ A}, pc = 1.5 \text{ GeV}$

Resonance analysis of SIS - 18

Use resonance strengths from analytic model to find relevant resonances.



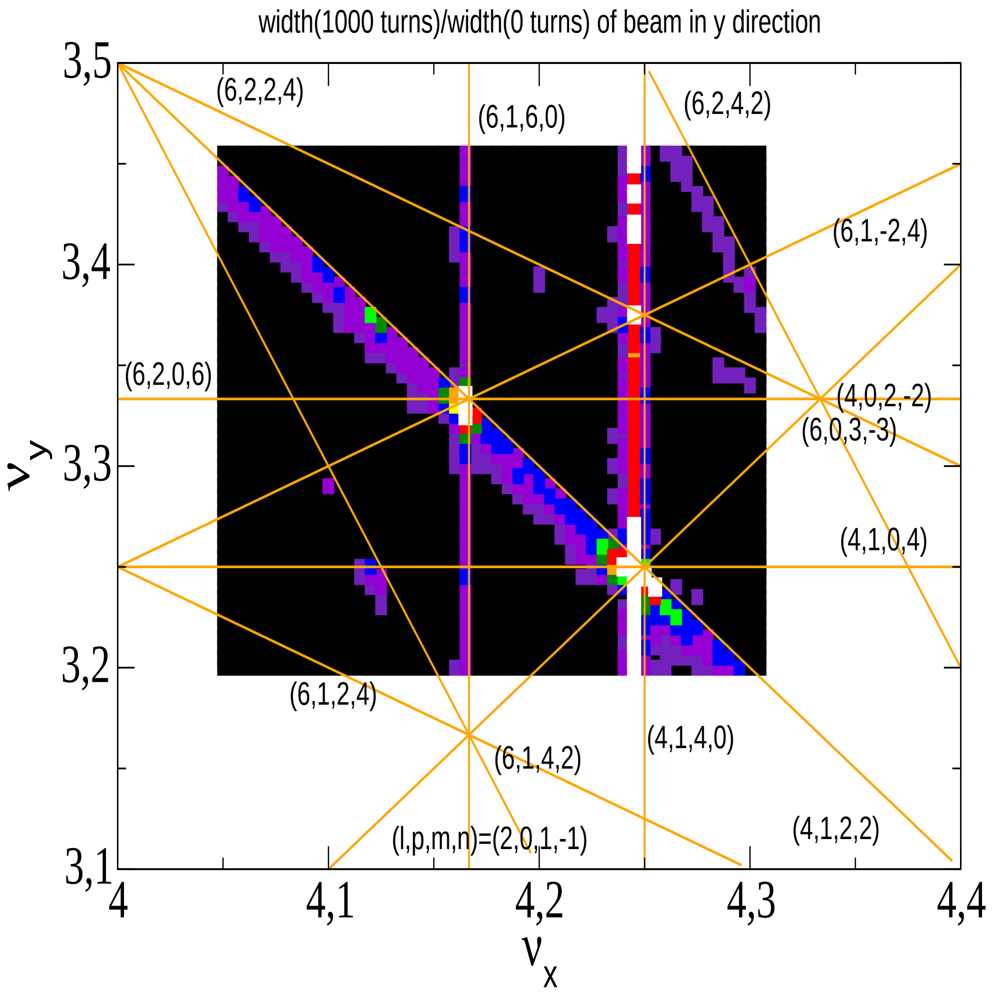
- **Flat top electron current profile (error function)**
- **Only resonances with even order appear**
- **Resonance strength strongly decreases with increasing order**

Figure from V. Ziemann, TSL-note 2004-60

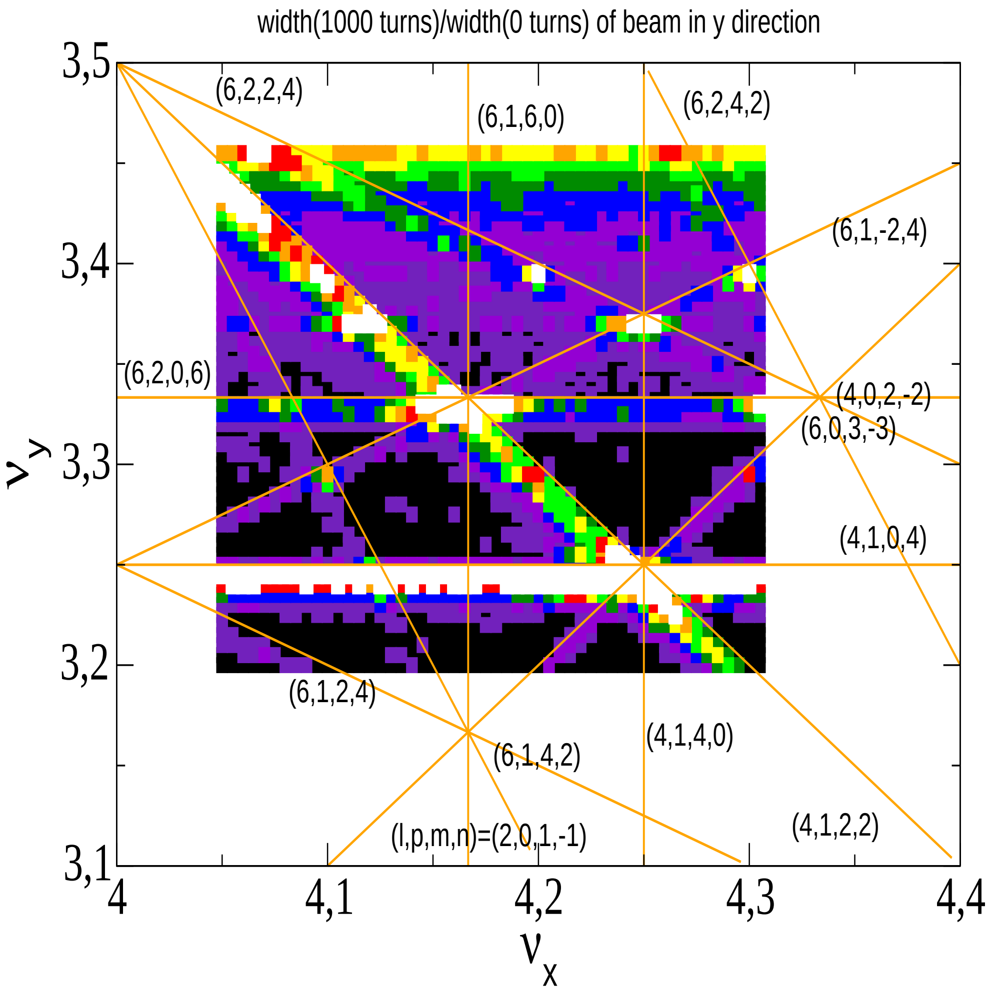
Resonance analysis of SIS - 18, $\nu_x - \nu_y$ – scan

Relative beam width $w_{\mathrm{rel}} = w_{\mathrm{fin}}/w_{\mathrm{ini}}$ depending on the tune

x direction

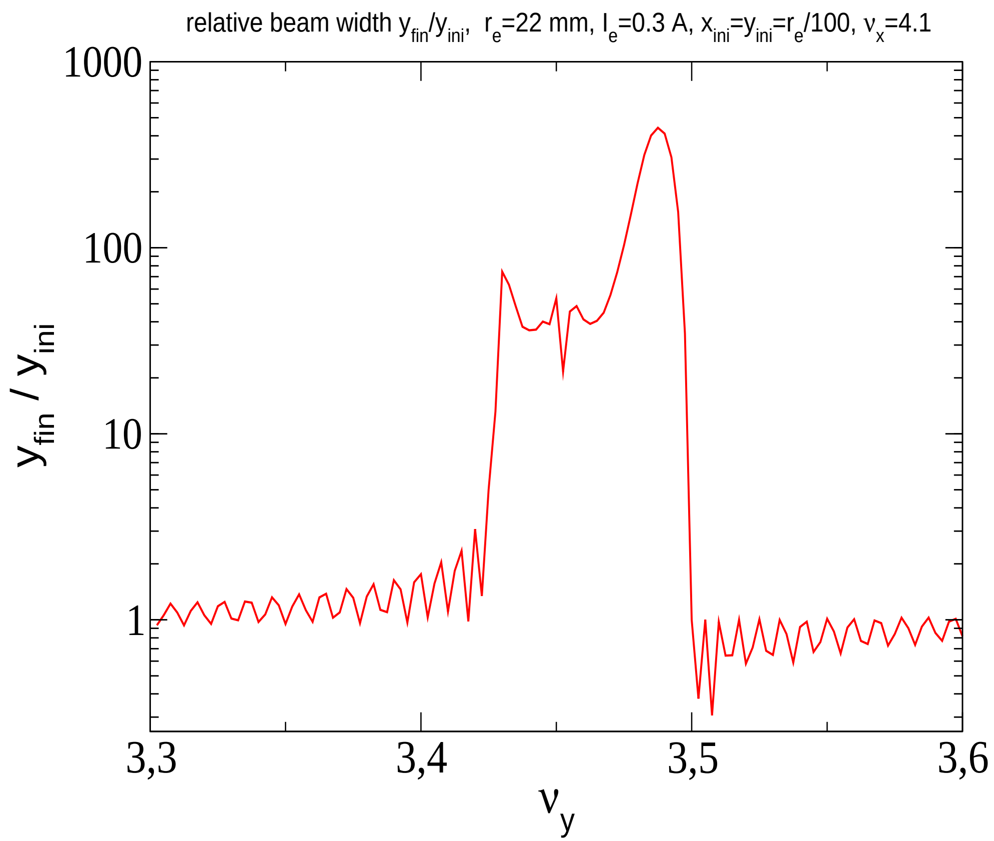


y direction



Resonance analysis of SIS - 18, ν_y – scan

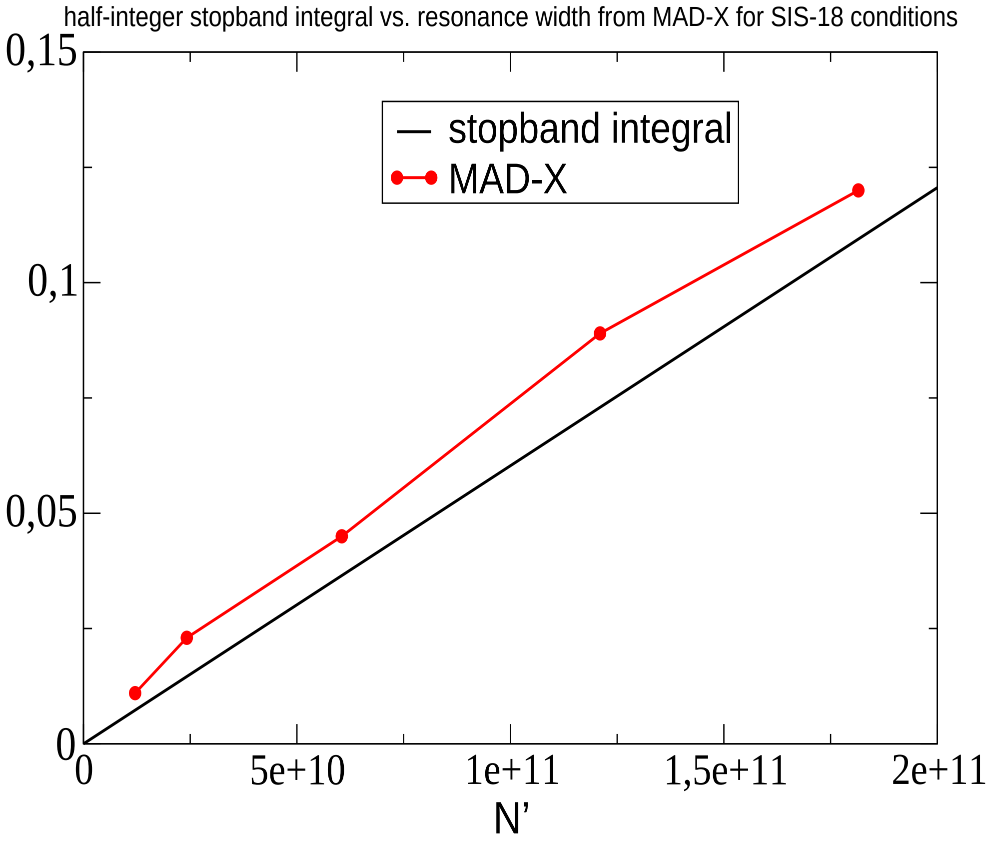
Relative beam width in y direction depending on the tune, $\nu_x = 4.1$



- $w_{\text{ini}} = b/100 \ll b$
  $\rightarrow$ **only quadrupole error due to electron beam.**
- **Near half integer tune, influence of the quadrupole error due to electron beam becomes significant.**
- **Increase of the beam width occurs only below half integer tune.**

Resonance analysis of SIS - 18, resonance width

Width of half integer resonance vs. half-integer stopband integral



- **half-integer stopband integral**

$$J_p = \frac{1}{2\pi} \oint \hat{\beta}\ k(s)\ e^{-ip\phi} ds$$

- **denotes the tune range, where**

$$\left| \frac{\Delta \hat{\beta}}{\hat{\beta}} \right| = 1,$$

i.e. beam size increases by a factor $\sqrt{2}$

Summary and outlook

- beambeam element has been modified to make it usable for the description of an electron cooler
- Modified element has been tested, results have been compared to those of analytic model with good agreement
- First application: Calculation of resonances driven by the electron cooler in SIS - 18
  - major reonances could be identified
  - results seem reasonable
- Possible experiments being under discussion:
  - measurement of resonances
  - Measurement of non-linear tunes, i.e. depending on betatron amplitude