1 Scaling of the Diffusive Aperture with the Wire Current

Passing the wire, a particle at vertical amplitude y receives approximately the kick:

$$\Delta y' = C \frac{I}{y-d} , \tag{1}$$

where $C \equiv (2r_p l_w)/(\gamma e c)$ with l_w the length of the wire, I the wire current, and d the distance. Here we have assumed that $x = 0$ for simplicity.

The tune shift at zero-amplitude ($y = 0$) is obtained from

$$\begin{aligned} \Delta Q_y &= \frac{\beta}{4\pi} \frac{d\Delta y'}{dy} \\ &= -C \frac{\beta}{4\pi} \frac{I}{d^2} , \end{aligned} \tag{2}$$

which reveals a scaling $I \propto d^2$. The sign of I is positive, if the force of the other wire is repelling. For a repelling force, this tune shift is negative.

We next look at the detuning with amplitude. It is

$$\begin{aligned} \frac{\Delta Q_y}{(\Delta J)/d^2} &= \frac{\beta^2}{32\pi} \left[\frac{d^3(\Delta y')}{dy^3} \right]_{y=0} d^2 \\ &= -\frac{3}{16\pi} \frac{C I \beta^2}{d^2} , \end{aligned} \tag{3}$$

where J denotes the action, which scales as d^2. A constant relative tune shift with amplitude is obtained for the same scaling $I \propto d^2$ as the constant zero-amplitude tune shift.

The scaling $I \propto d^2$ is also found by applying Chirikov's resonance overlap criterion for the onset of global chaos to the $1/r$ force (see Eq. (9) in [1]).

Figure 1 shows a simulation result using the weak-strong simulation code DIFF. The nominal beam-wire distance at a proton momentum of 26 GeV/c was taken to be 24.23 mm, corresponding to 9.5σ for a normalized emittance of 3.75 μm and $\beta = 50$ m (the rms beam size is $\sigma = 2.55$ mm). If the beam-wire distance is decreased by a factor of 2, the same relative diffusive aperture is obtained for a quarter of the wire current. This is consistent with the expected scaling. We also note that a significant effect is observed for a large range of wire currents, with rather small changes in the diffusive aperture.

Figure 2 displays some example simulation results for a beam-wire separation of 24.23 mm. The sudden jumps in the action spread are characteristic of the chaotic motion of individual particles. It is not at all clear that the resulting macroscopic behavior should be well described by a diffusion equation.

References

[1] Y. Papaphilippou, F. Zimmermann, "Estimates of Diffusion due to Long-Range Beam-Beam Collisions," Phys. Rev. ST Accel. Beams 5, 074001 (2002).

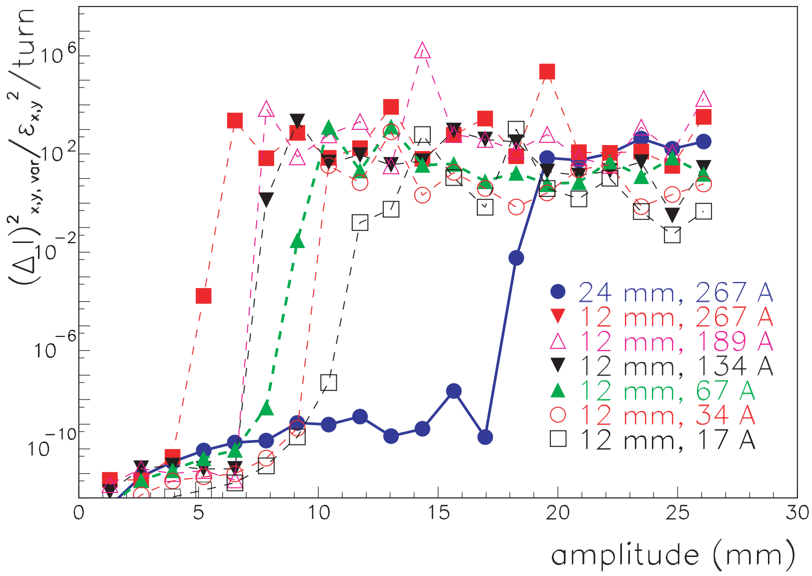

Figure 1: Diffusion rate as a function of start amplitude (at $\beta = 50$ m) for two different beam-wire distances and several different wire excitations as indicated. The effective length of the wire is taken to be 1.2 m. The simulations were performed for $Q_x = 0.1781$ and $Q_y = 0.1508$.

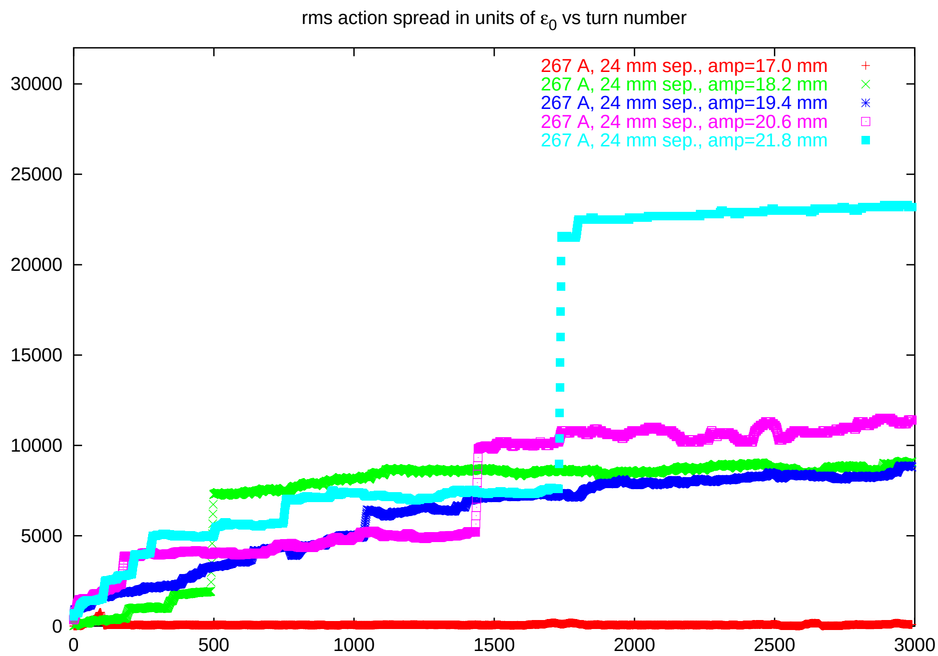

Figure 2: Increase in the rms action spread for several groups of particles launched at the same initial amplitude as a function of time, for a beam-wire distance of 24.23 mm. The various curves refer to different start amplitudes. The simulations were performed for $Q_x = 0.1781$ and $Q_y = 0.1508$.