Amplitude Diffusion due to Long-Range Collisions

- simulation model
- result from 1999 LHC project note for 4 IPs
- new (additional) simulations (?)
- inclined crossing

Weak-Strong Simulation Model

Y. Papaphilippou & F. Z., PRST-AB 2 104001, 1999.

The simulation study is performed in the spirit of John Irwin (SSC-223); it is 4 dimensional

We treat two (to four) IPs, symmetrically spaced around the ring, with horizontal and/or vertical crossing.

At each IP we apply a series of kicks representing:

- long-range collisions (incoming side)
- head-on collision
- long-range collisions (outgoing side)

Triplet errors can optionally be included.

Model for Long-Range Interactions

All parasitic collisions (n_{par}) on one side of the IP are lumped. The kick is approximately expressed as a change in the IP coordinate (while the IP angle stays unchanged). For horizontal crossing:

$$\Delta x = n_{par} \, \frac{2 r_p N_b}{\gamma} \, \left[\frac{x' + \theta_c}{\theta_t^2} \left(1 - e^{-\frac{\theta_t^2}{2\sigma^2}} \right) - \frac{1}{\theta_c} \left(1 - e^{-\frac{\theta_c^2}{2\sigma^2}} \right) \right]$$

$$\Delta y = n_{par} \, \frac{2 r_p N_b}{\gamma} \, \left[\frac{y'}{\theta_t^2} \left(1 - e^{-\frac{\theta_t^2}{2\sigma^2}} \right) \right]$$

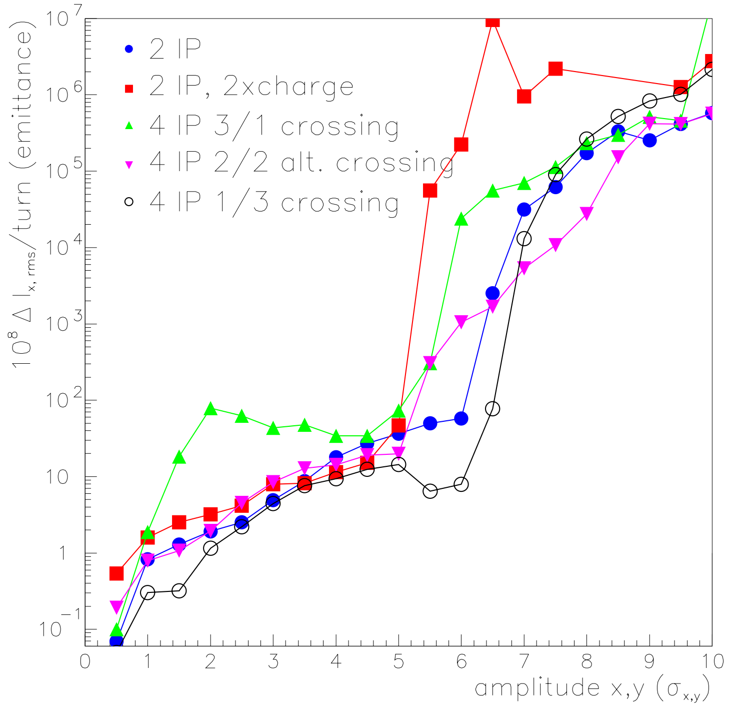
where $\theta_t \equiv \left((x' + \theta_x)^2 + y'^2 \right)^{1/2}$. Effective number of parasitic crossings per side $n_{par} \approx 15$. The kick is the same on both sides of the IP. The vertical crossing is treated analogously.

Amplitude Diffusion

Following Irwin (SSC-223), we study the evolution of the action variance for a group of particles, launched at the same values of the two transverse actions (located on a circle in horizontal or vertical phase space with random initial phase).

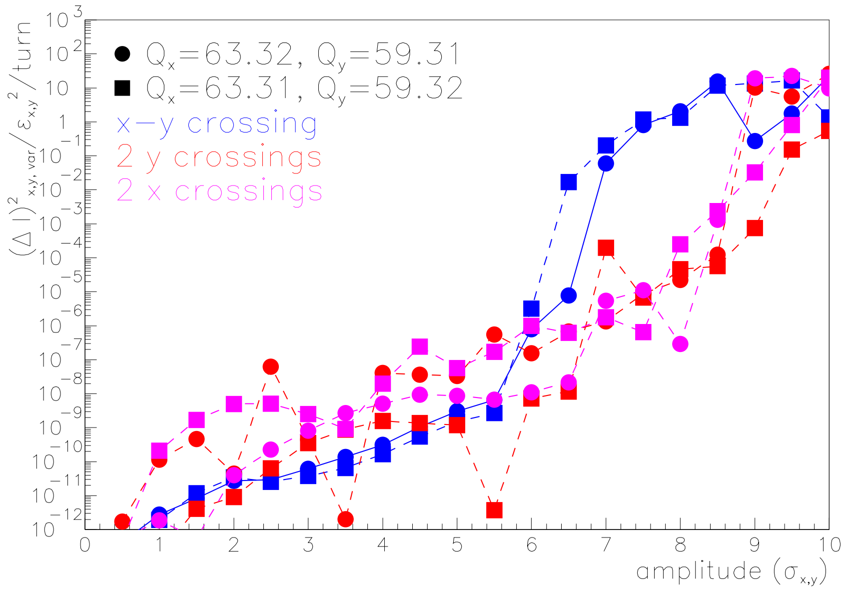
To suppress short-time fluctuations, *e.g.*, caused by static deformations of the invariant tori in phase space due to resonances, we compute the running average over 1000 turns of the rms action spread.

Diffusion for different no.'s of IPs & crossings



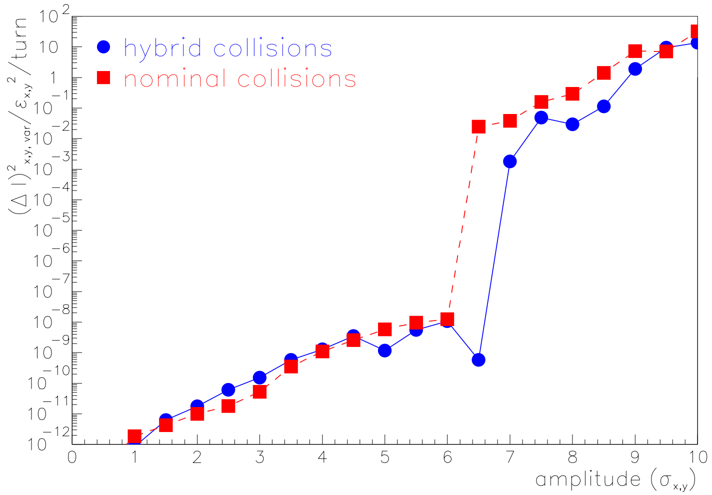
Simulated action diffusion rates vs. starting amplitude, for different number of IPs, parasitic collision points (or charge), and crossing schemes; $Q_x = 0.31$ and $Q_y = 0.32$; other parameters: $\epsilon_N = 3.75\ \mu$m, energy 450 GeV, $n_{\rm sep} = 10$, and $n_{\rm par} = 18$. (F.Z., LHC Project Note 250).

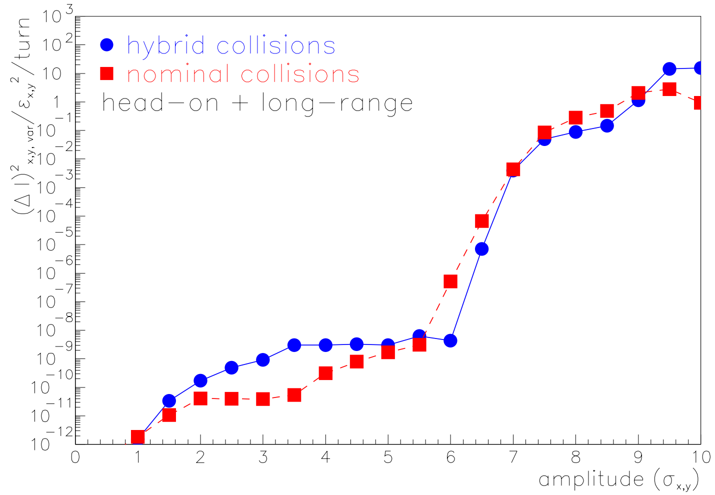
Diffusion for different no.'s of IPs & crossings



Simulated action diffusion rates vs. starting amplitude, for 2 IPs with head-on & long-range collision, considering 3 crossing schemes for 2 different bare tunes; other parameters: $\epsilon_N = 3.75\ \mu$m, energy 450 GeV, $n_{\rm sep} = 9.5$, $N_b = 10^{11}$, and $n_{\rm par} = 15$. Very preliminary!

X-Y Crossing vs. Inclined Crossing (45° & 135°)





Simulated diffusion rate vs. start amplitude with $x-y$ alternating crossing and with inclined collisions; $\beta^{*}=0.5$ m, $\theta^{*}_{x,y}=31.7$ μrad, $\theta_c=300$ μrad; nominal LHC. left: long-range collisions only; right: long-range & head-on collisions (Y.P. & F.Z., PRST-AB 2 104001, 1999)

Closest Tune Approach (Coupling Strength)

$$|\kappa_-| = \left| \frac{1}{2\pi} \oint ds\ K_s \sqrt{\beta_x \beta_y} e^{i(\phi_x - \phi_y - (Q_x - Q_y - q)\frac{2\pi s}{L}} \right|$$

or, with two IPs,

$$|\kappa_-| = k\,|1 + \exp(i(\Delta\phi_x - \Delta\phi_y - \Delta Q\pi))| \quad \text{with}$$

$$k = \frac{n_{\rm par} K_s \beta_s}{2\pi} = \frac{n_{\rm par} N_b r_p}{\pi(\gamma\epsilon_y) n_{\rm sep}^2}$$

where $n_{\rm sep}$ separation in σ, $n_{\rm par}$ no. of parasitic collisions around each IP, $\Delta\phi_{x,y}$ phase advance between the two IPs. For the LHC: $k \approx 0.005$. With 45° and 135° coupling is zero, for $\Delta\phi_x - \Delta\phi_y - \Delta Q\pi = n2\pi$. Another (better?) approach: choose $\Delta\phi_x \& \Delta\phi_y = n2\pi$.

Conclusions

- 'diffusive aperture' can vary $> 1\ \sigma$ depending on working point and crossing scheme
- x-y crossing desensitizes PACMAN bunches, by equalizing their working point
- inclined crossing is an interesting option; consequences of coupling need to be examined
- more simulations & complete study with SIXTRACK desirable