Weak-strong Simulation Studies for the LHC Long-Range Beam-Beam Compensation

1 Model

head-on, long-range, wire compensation

2 Results

tune footprints, action diffusion vs. time, various linear imperfections

3 Conclusion

1. Model

The simulation study was performed in the spirit of John Irwin (SSC-223, 1989) and article PRST-AB 2 104001 (1999) by F.Z. & Yannis Papaphilippou; it is 4 dimensional, with optional tune modulation.

We treat two IPs, one with horizontal crossing, the other with vertical.

At each IP we apply a series of 3 kicks representing:

- long-range collisions and wire compensation (incoming side)
- head-on collision
- long-range collisions and wire compensation (outgoing side)

parameter	symbol	value
number of particles per bunch	N_b	1.05×10^{11}
beam energy	E_b	7 TeV
rms beam size at IP	$\sigma^*_{x,y}$	16μm
rms divergence at IP	$\theta^*_{x,y}$	31.7 μrad
IP beta function	$\beta^*_{x,y}$	50 cm
full crossing angle	θ_c	300 μrad
number of collision points	n_{IP}	≥ 2
number of bunches per beam	n_b	2835
bunch spacing	L_{sep}	7.48 m
beam-beam parameter	ξ	0.00342
revolution frequency	f_{rev}	11.25 kHz

I. Head-On Collision

For round Gaussian beams:

$$\Delta x' = \frac{2 r_p N_b}{\gamma} \frac{x}{r^2} \left(1 - e^{-\frac{r^2}{2\sigma^{*\ 2}}} \right) \quad (1)$$

$$\Delta y' = \frac{2 r_p N_b}{\gamma} \frac{y}{r^2} \left(1 - e^{-\frac{r^2}{2\sigma^{*\ 2}}} \right) \quad (2)$$

where $\sigma \equiv \sigma_x = \sigma_y$; $r = \sqrt{x^2 + y^2}$ is the radial distance to the origin, r_p the classical proton radius, γ the Lorentz factor, and N_b the bunch population.

II. Long-Range Interactions

All parasitic collisions (n_{par}) on one side of the IP are lumped. The kick is approximately expressed as a change in the IP coordinate (while the IP angle stays unchanged). For horizontal crossing:

$$\Delta x = n_{par} \, \frac{2 r_p N_b}{\gamma} \left[\frac{x' + \theta_c}{\theta_t^2} \left(1 - e^{-\frac{\theta_t^2}{2\sigma^2}} \right) - \frac{1}{\theta_c} \left(1 - e^{-\frac{\theta_c^2}{2\sigma^2}} \right) \right]$$

$$\Delta y = n_{par} \, \frac{2 r_p N_b}{\gamma} \, \frac{y'}{\theta_t^2} \left(1 - e^{-\frac{\theta_t^2}{2\sigma^2}} \right)$$

where $\theta_t \equiv \left((x' + \theta_c)^2 + y'^2 \right)^{1/2}$. Effective number of parasitic crossings per side $n_{par} \approx 18$. The kick is the same on both sides of the IP. The vertical crossing is treated in complete analogy.

III. Wire Compensation

For a horizontal crossing, the effect of a thin wire is represented as:

$$
\begin{aligned}
\Delta x &= \frac{\mu_0 I_w l_w}{2\pi(B\rho)} \left[\frac{x' + \theta_{c,w} \pm \phi_x x/\beta_x^*}{\theta_{tw}^2} - \frac{1}{\theta_{c,w}} \right] \\
\Delta x' &= -(\pm 1)\phi_x \; \Delta x/\beta_x^* \\
\Delta y &= \frac{\mu_0 I_w l_w}{2\pi(B\rho)} \; \frac{y' \pm \phi_y y/\beta_y^*}{\theta_{tw}^2} \\
\Delta y' &= -(\pm 1)\phi_y \; \Delta y/\beta_y^*
\end{aligned}
$$

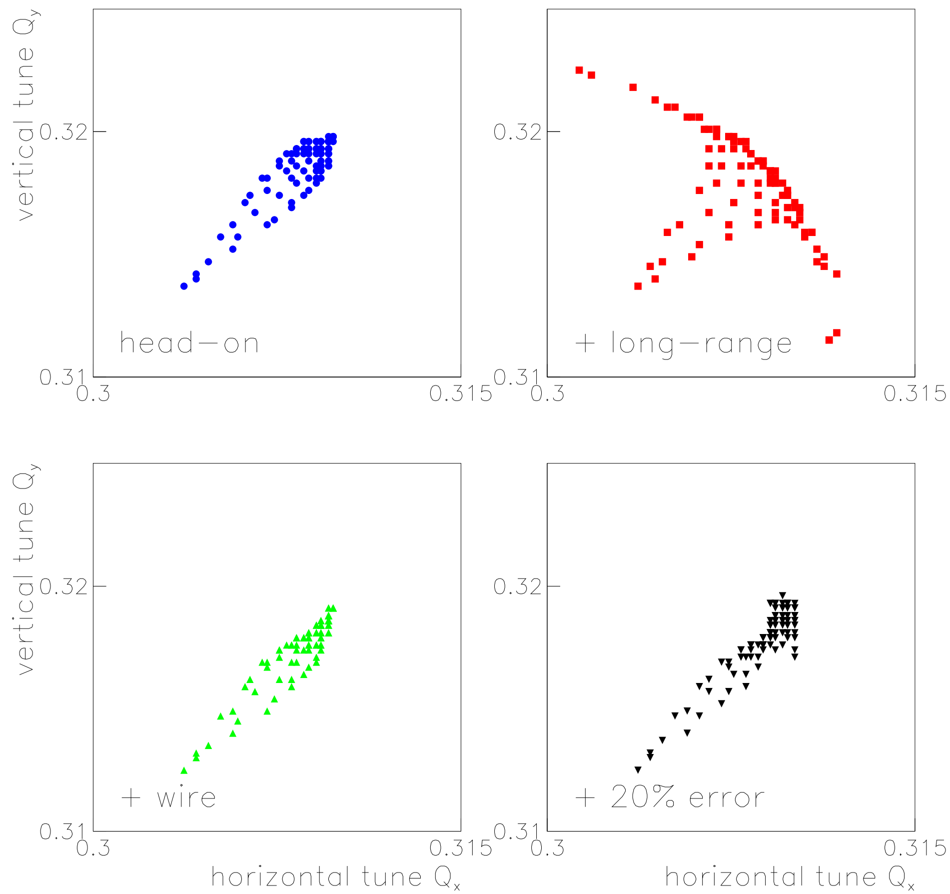
where $\theta_{tw} \equiv \left((x' + \theta_{c,w} \pm \phi_x x/\beta_x^*)^2 + (y \pm \phi_y y/\beta_y^*)^2\right)^{1/2}$, and for perfect compensation $I_w = 4\pi(B\rho)N_b r_p n_{par}/(\mu_0 \gamma l_w)$. The $\pm$ signs refer to the two sides of the IP. The vertical crossing is treated in complete analogy.

IV. Errors

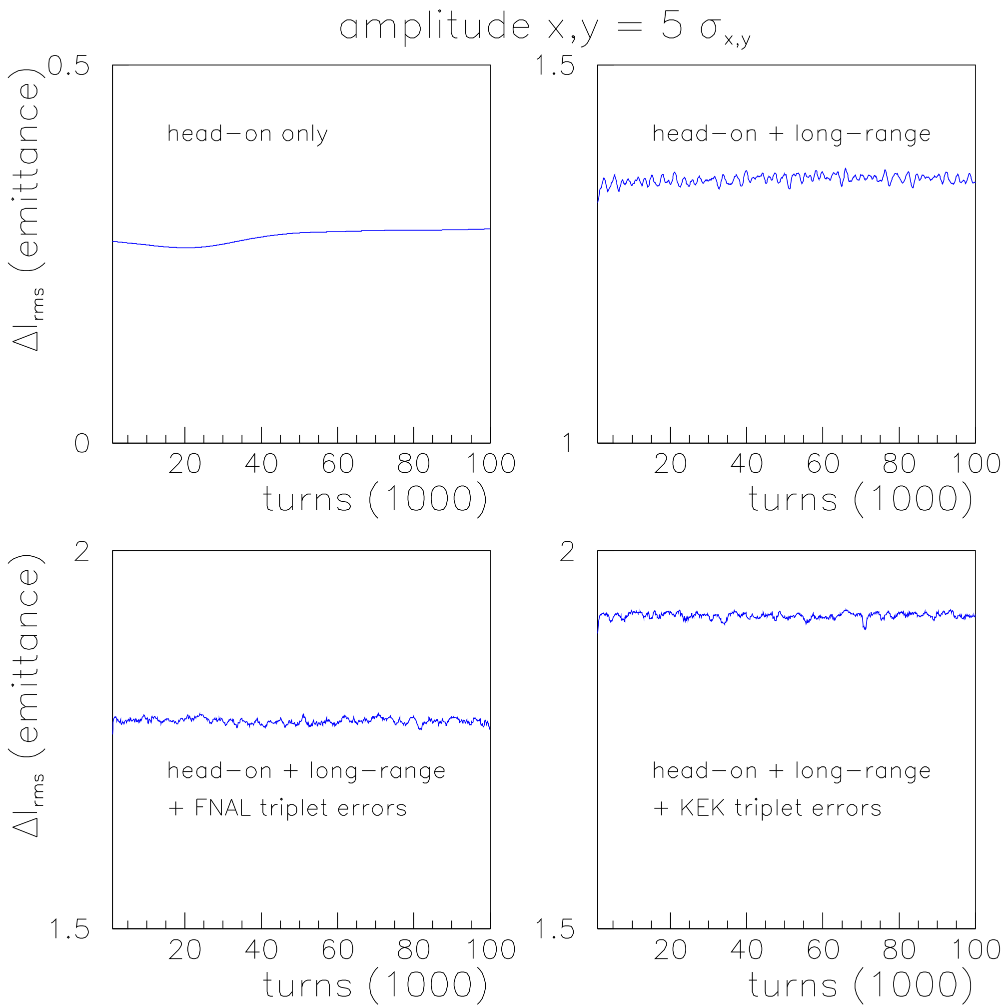
four types were considered:

- simultaneous symmetric betatron phase shift $\phi_{x,y}$ on both sides of each IP
- wire strength error
- wire-beam distance error
- betatron phase shift $\phi_{x,y}$ with only one wire per IP

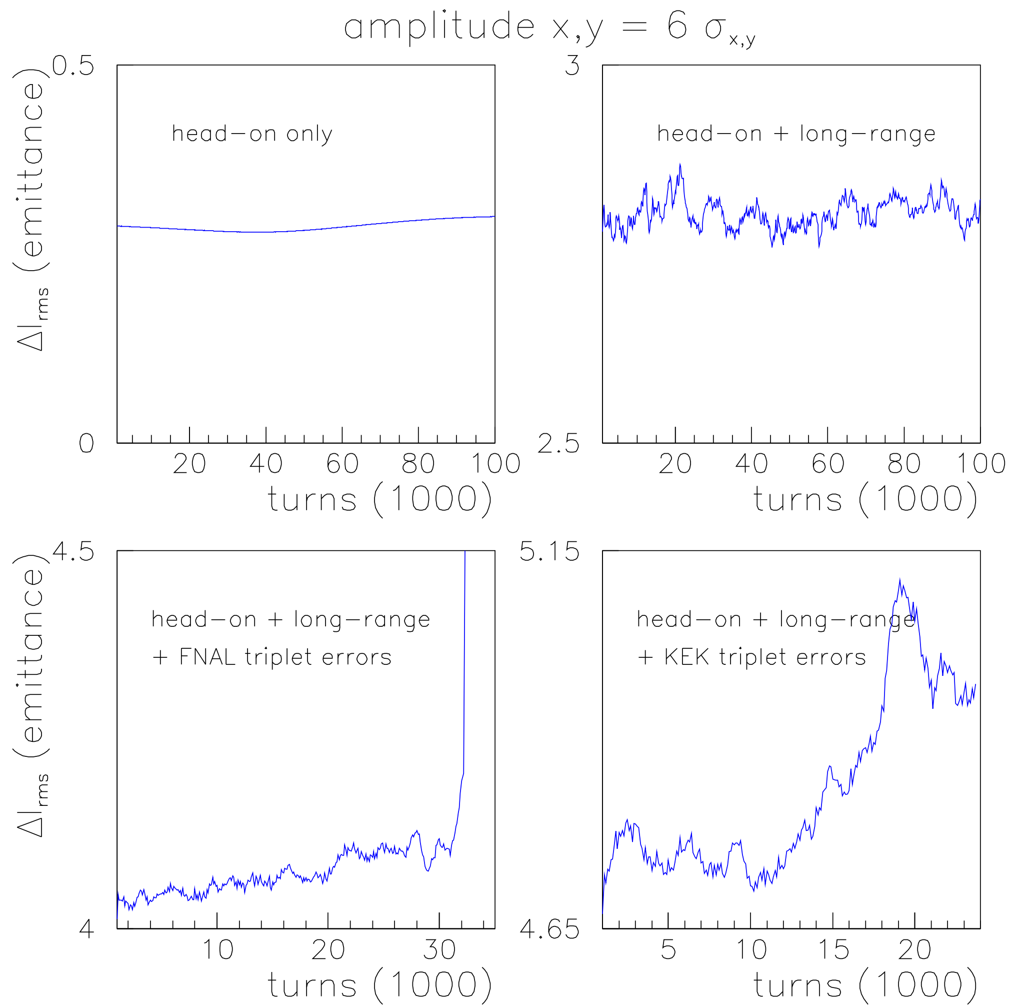
2. Results



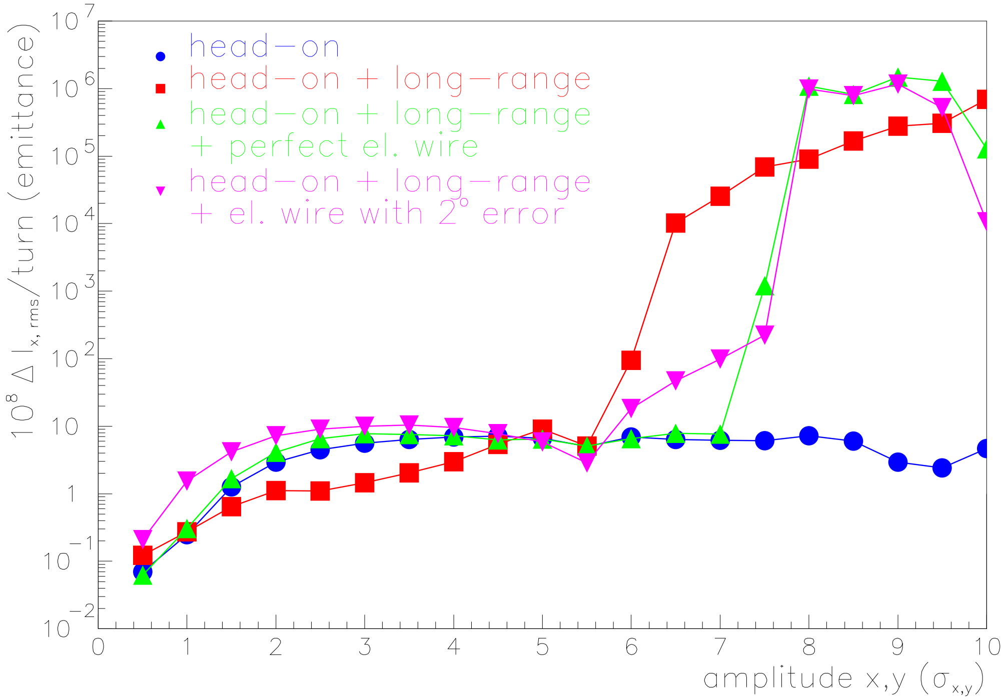
Tune footprints for various cases, for initial horizontal and vertical amplitudes extending to $7\sigma_{x,y}$.



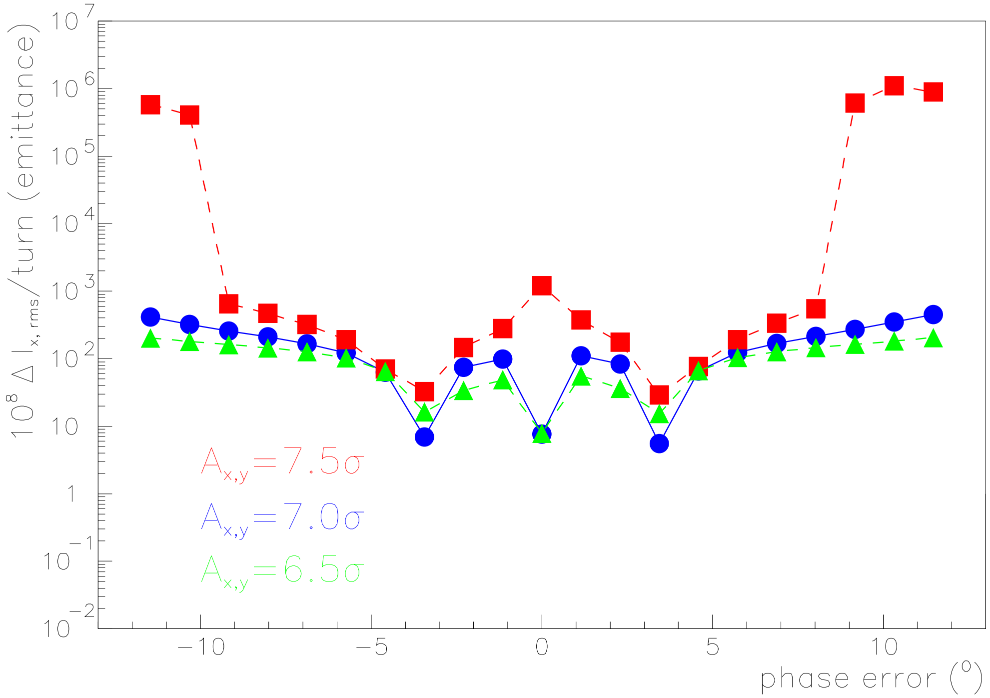
The variance in action for a group of 100 particles, launched at 5 $\sigma_{x,y}$, as a function of turn number. The motion is stable.



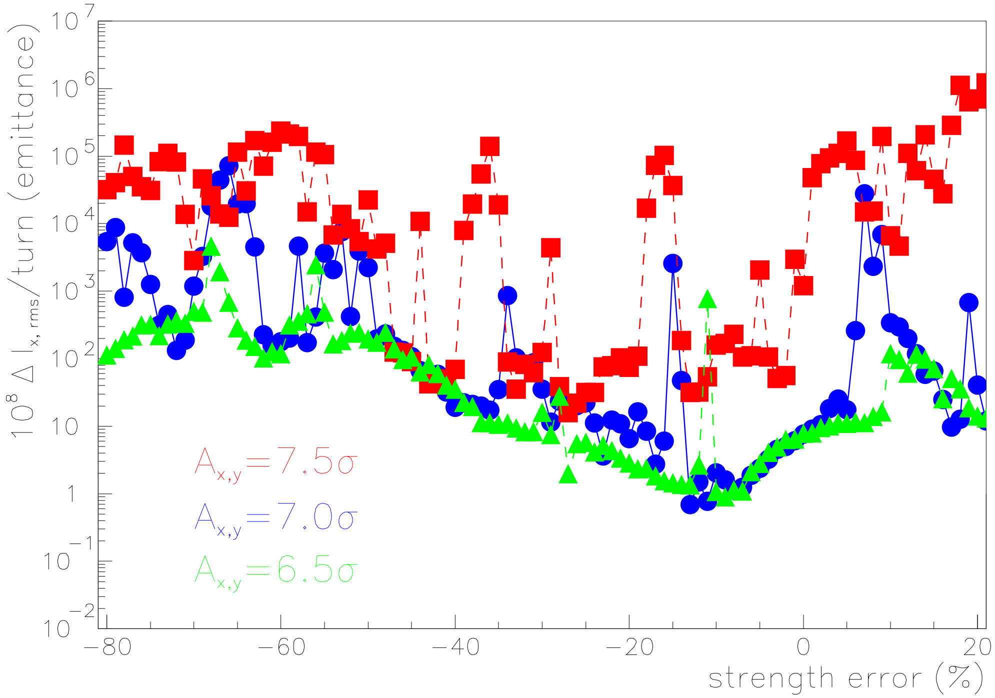
The variance in action for a group of 100 particles, launched at 6 $\sigma_{x,y}$, as a function of turn number. Some particles are lost.



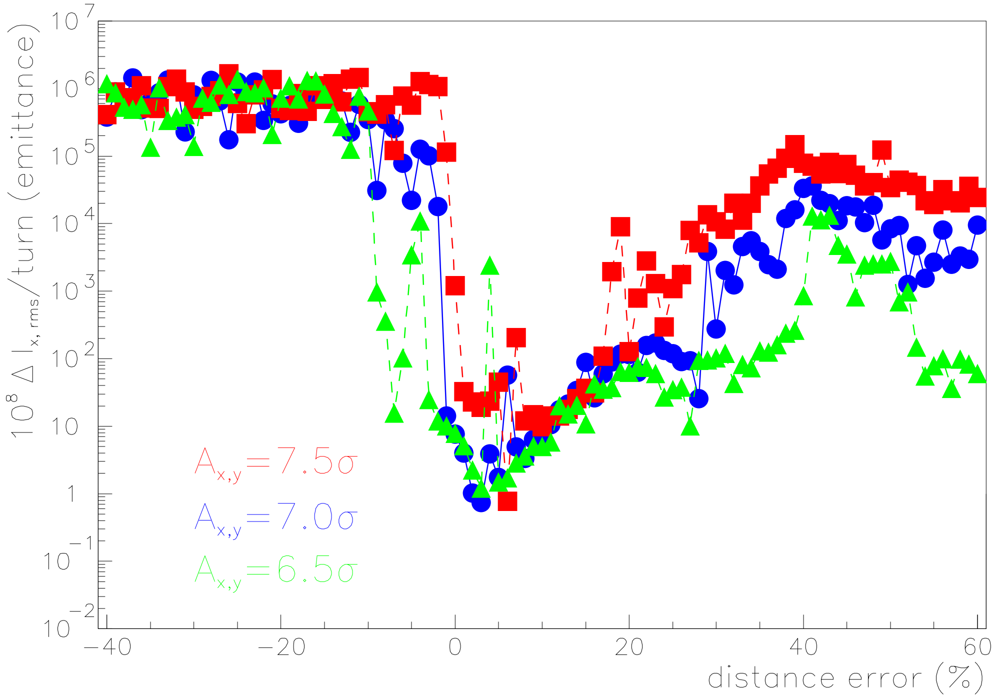
The diffusion per turn as a function of the start amplitude. Different cases are compared.



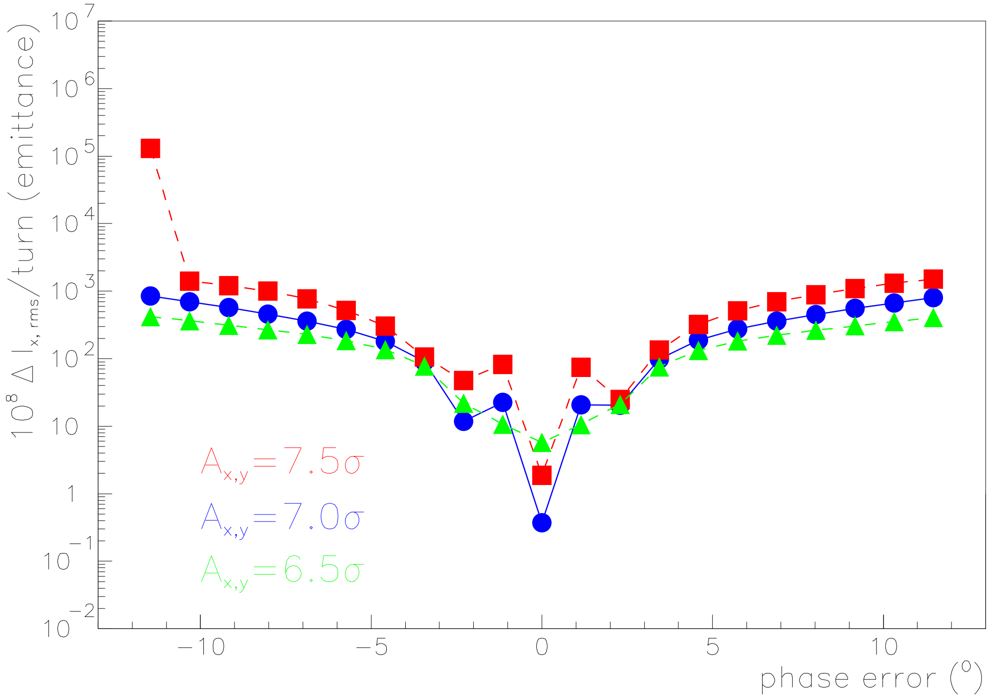
Variation of diffusion rate with symmetric betatron phase error at various amplitudes.



Variation of diffusion rate with wire strength error at various amplitudes.



Variation of diffusion rate with wire position error at various amplitudes.



Variation of diffusion rate with betatron phase error at various amplitudes, if there is a compensating wire only on one side of each IP.

3. Conclusions

- wire compensation works well
- tolerance to phase errors $\sim 10^\circ$
- tolerance to strength errors -40% to $+20\%$
- tolerance to distance errors -5% to $+40\%$