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Principle of a Correction of the Long-Range Beam-Beam Effect in LHC using Electromagnetic Lenses

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Summary

The beams in LHC collide head-on in at most four experimental points. Due to the small bunch spacing, the beams experience in addition more than one hundred 'near-misses' on either side of the collision points. The transverse beam separation at these places, with a crossing angle of $\pm 150\mu$rad is in the range of 7 to 13 σ. The residual interactions between the two beams (the so-called 'long-range' interactions) are partly compensated by alternating the plane of the crossing angle in IP1 and IP5. It is however more and more recognized that the non-linear part of these interactions which are not compensated by the alternating crossing remains the dominant mechanism of beam blow-up or beam loss for amplitudes beyond 6σ. It may thus cause background to the experiments and a reduction of the beam lifetime. We show in this note that, contrary to the case of the head-on beam-beam interaction, a simple non linear model of the long-range interactions can be devised. This model suggests a rather simple correction principle by electro-magnetic lenses, basically wires, which correct with a good accuracy simultaneously the linear and non-linear perturbations. The correction of the average perturbation over all the bunches seems not demanding. An exact correction of the so-called PACMAN bunches may be done at a frequency an order of magnitude lower than the bunch frequency and is being evaluated.

1 Introduction

With a 25 ns bunch spacing, there would be 30 head-on collisions per experimental insertion. The beams therefore cross at an angle. The low-β quadrupoles being common to the two LHC rings, this angle cannot be large as both beams must fit into a single aperture. As a result, the perturbations due to the electro-magnetic field of one beam acting onto the other (long-range beam-beam effect, LR in short) cannot be made negligible compared to the head-on perturbation.

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The criterion chosen to evaluate the beam-beam effect and calculate the LHC performance is the tune spread (i.e. extent of the tune footprint) caused by the beam-beam forces. The beam intensities are chosen such that the tune footprint just fits between resonance lines experimentally identified to cause a high background or a reduction of the lifetime. This criterion was successfully tested in the Sp$\bar{p}$S. For LHC, the same criterion is used but including the significant effect of the LR interactions in addition to the head-on beam-beam effect.

At the occasion of tracking studies, a strong perturbation of the single particle dynamics associated with the LR beam-beam effect has been identified even-though the above-mentioned criterion was fulfilled. By tracking at collision energy (Chou and Ritson [1]) and at injection energy (Miles [2]), it was discovered that the LR interactions decrease the dynamic aperture in an unacceptable way. The crossing angle was increased by 50% in collision and close to 100% at injection. The particle amplitude for the calculation of the footprint criterion was increased from 4σ to 6σ to be consistent with the tracking results. Recently Sen [3], Papaphilippou and Zimmermann [4] followed by Grote, Leunissen and Schmidt [5], by studying more refined observables than the dynamic aperture found abnormal amplitude growth or onset of chaos below the dynamic aperture that could be attributed to LR interactions. The latter act as the dominant destabilizing mechanism, with a diffusion in amplitude and tune much larger than that due to the head-on beam-beam effect. At large amplitudes, the triplet non-linearities, though corrected, cause the motion to become unstable.

There is therefore a strong incentive to reduce the strength of the LR beam-beam interactions. The crossing angle cannot be much increased. The alternating crossing angles in IP1 and IP5 [6] which is part of the nominal design, minimize the linear shift of the tune but does not decrease in general the non-linear forces.

Compensating the beam-beam effect by electro-magnetic fields was mentioned as a subject for research in the conclusions of the ICFA workshop on the beam-beam effect in Novossibirsk, 1989. A beam-beam lens based on an auxiliary electron beam is being studied and constructed at Fermilab [8].

We consider in this study a correction scheme specialized for the LR interactions. Although very simple in its principle, it can cope with the linear and non-linear components of the incoherent LR effects.

2 The Long-Range Beam-Beam Interactions in LHC

The long-range beam-beam interactions occurring in IR2 and IR8 are weak as compared to IR1 and IR5 [7], due to the larger beam separation. In this note, we neglect them and only consider the case of the high-luminosity insertions. With the aim of a purely local correction, we further concentrate on only one of the two identical insertions.

2.1 Terminology

Following the tradition, the sample particle of one beam is called the weak beam, with W as a subscript. It suffers from the perturbation of the second 'strong' beam, denoted by S.

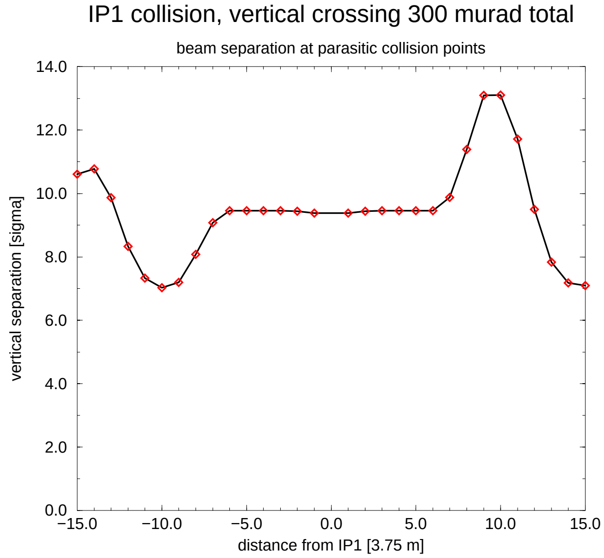


Figure 1: Beam separation in the crossing angle plane of IR1 in σ_W's from [9]

2.2 Layout and Optics

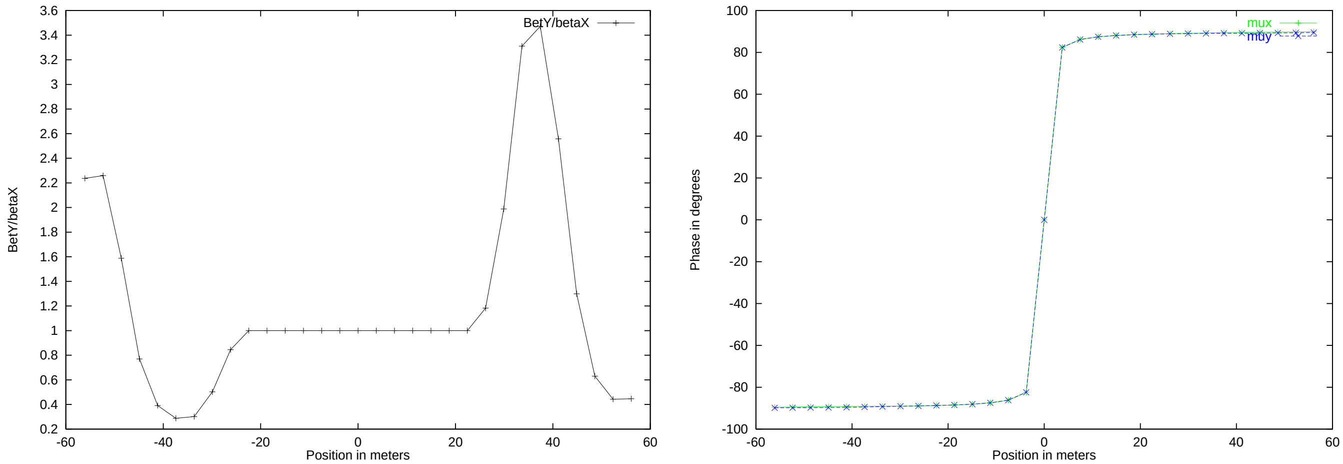
The beam separation in IR1 for the nominal crossing angle of $\pm 150\mu$rad is shown on Figure 1. In IR5 the separation is the same but in the horizontal plane. Some relevant optical functions at the LR interactions are shown on Figure 2 for the 'weak' beam. The nominal optics in the machine section common to the two beams is strictly anti-symmetrical. The parameters of the strong beam are simply those of the weak beam observed at a symmetrical azimuth with respect to the IP (or at the same azimuth but inverting the horizontal and vertical planes).

Figure 2 (b) shows that the 15 kicks experienced by the beam on each side of an IP are lumped in betatron phase (the average and rms phases are 88.45° and 1.95°). Furthermore, the sign of the kicks changes from one side of the IP to the other with the sign of the separation and so does the amplitude of the particles due to the betatron phase shift of almost 180°. The perturbations of the 30 kicks are therefore almost additive except between the kicks, i.e. at the crossing point.

Figure 2 (a) shows that the beams are round to within 10% in about 60% of the cases. The largest aspect ratio is about 1.8. This ellipsity is to be compared to a beam separation of at least 7 in the same units. We therefore assume in the analysis round beams. The numerical calculations will be performed with the exact beam aspect ratios.

2.3 Instantaneous Beam Current in LHC

The nominal number of particles per bunch is 1.1 10^{11} as of LHC version 6.0. The nominal rms bunch length σ_s is 0.257 ns or 7.7 cm. Assuming that the total bunch length is given by $\sqrt{2\pi}\sigma_s$, the instantaneous beam current is 27.36 A. For the ultimate performance, the beam current is increased by 60% and reaches 43.77 A.



(a) (b)

Figure 2: Ratio of the β-functions and phase shift with respect to the IP in IR5

2.4 The Magnetic Field of the Beam

For ultra-relativistic counter-rotating beams, it is well known that the forces exerted by the electric and magnetic fields are equal to a very good approximation, see e.g. [10]. The relative error is $(1-\beta^2) = 1/\gamma^2$, i.e. $2\ 10^{-8}$. The total force may be derived from an effective magnetic field with an intensity doubled.

The perturbation however is measured by the integration of the force over the interaction time. As the two beams move in opposite directions, the interaction time is half the bunch length. The factors of two cancel out and the kick will not be different from the kick due to the magnetic field of non relativistic moving charges. In the following, we will therefore only consider the true magnetic fields in the laboratory frame (not the effective one) and compute its integral over the full length of the strong beam.

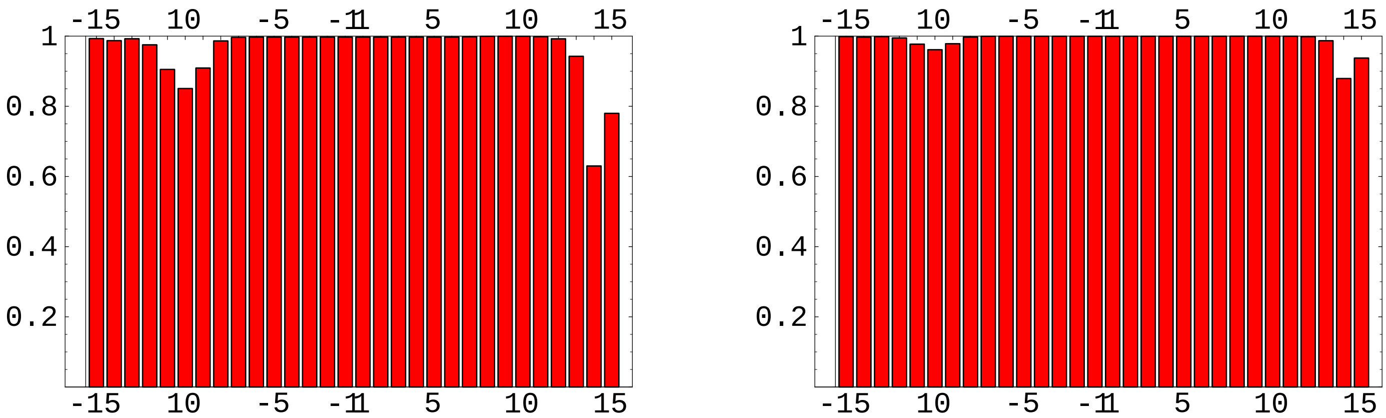
For a Gaussian charge distribution, the expression of the magnetic field in the laboratory frame is, e.g. [11]:

$$B_\theta = \frac{\mu_0}{2\pi}\ \frac{I_b}{r}\ (e^{-r^2/2\sigma_S^2} - 1) \tag{1}$$

I_b is the beam current, r the distance between the center of the strong beam and the test particle of the weak beam, σ_S the transverse beam size of the strong beam assumed **cylindrical** which, as we saw, seems a reasonable assumption for LHC.

The dependence of the field B_θ on the relative particle position r is involved in general. However, for the LR interactions, the present nominal LHC parameters are such that the exponential term in Eq. 1 is small. We show in the following that it can be neglected.

The largest amplitude of the betatron oscillation allowed by the collimation system is $6\sigma_W$. In the worst case where this amplitude is fully in the plane of the beam separation, the distance r is computed for each LR interactions and the corresponding bracketed exponential term in Eq. 1 is calculated (Figure 3(a)). On Figure 3(b), this term is further averaged over one betatron oscillation of the same extreme particle to give a less pessimistic picture of the



(a) worst case κ

(b) average η

Figure 3: exponential term in Eq. 1 versus the LR interaction position. (a) is the worst case, (b) is the kick averaged over the betatron oscillation.

perturbation. It can be seen that the impact of the exponential term is relatively small. In the worst case, summed over the 30 LR interactions, it amounts to 4% and is reduced to 1% when considering averaging over the betatron oscillation.

This approximate analysis shows that the strong beam may be rather accurately modelled by an electric current in a conductor producing the same $1/r$ dependence of the magnetic field. The long-range beam-beam effect appears, in this light, simpler than expected: all the 30 kicks experienced in one interaction region are almost additive and each of them can be modelled replacing the strong beam by an equivalent electric current in a wire. This opens the possibility, at least in principle, to a lumped correction using again an electric current in a wire suitably positioned and counter-acting the LR perturbations.

2.5 The Magnetic Field of a $1/r$ Lens

Following figure 4 and after calculation, the magnetic field induced by a wire at position O' (r_0, ϕ) on the weak beam at position (x, y) is given, in Cartesian coordinates, by:

$$B_x = \frac{\mu_0 I_b}{2\pi} \frac{-y + r_0 \sin\phi}{r_0^2 + x^2 + y^2 - 2r_0(x\cos\phi + y\sin\phi)} \tag{2}$$

$$B_y = \frac{\mu_0 I_b}{2\pi} \frac{x + r_0 \cos\phi}{r_0^2 + x^2 + y^2 - 2r_0(x\cos\phi + y\sin\phi)} \tag{3}$$

By virtue of Ampere's law, this expression is independent of the wire diameter as long as it does not intercept the beam. For perturbative calculations, the multipole expansion can be convenient:

$$B_y + iB_x = \frac{\mu_0 I_b}{2\pi r_0} \sum_{n=1}^{\infty} (-\cos n\phi - i\sin n\phi) \frac{(x+iy)^{n-1}}{r_0^{n-1}} \tag{4}$$

$B_\theta(x = y = 0)$ appears naturally as the reference field and r_0 as the reference radius.

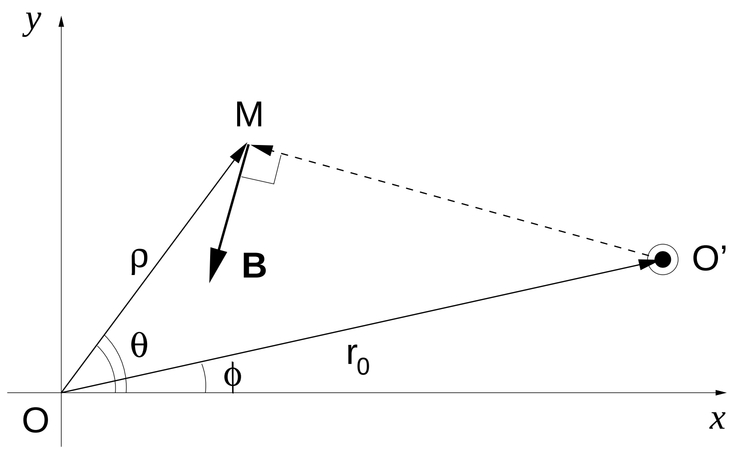


Figure 4: Coordinate system for the wire (O') and the particle position(M). O is the barycenter of the weak beam.

Figure 5 shows the decrease of the field perturbation $B_y(x = 6\sigma)$ due to a given multipole order. The wire is placed at 9.5σ (average separation between LR encounters) in the horizontal plane ($\phi = 0$). Unlike the case of the head-on collision, the convergence appears

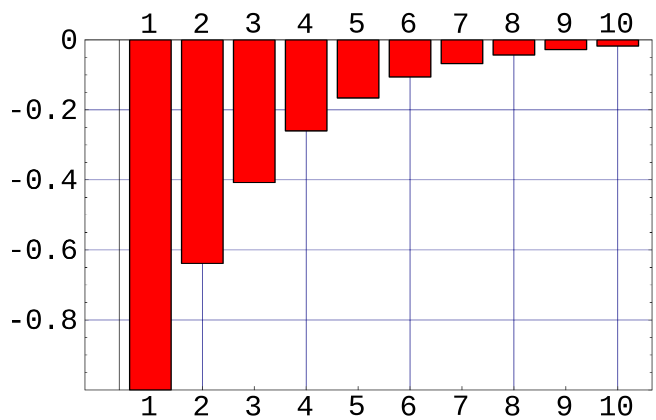


Figure 5: Relative field perturbation at 6σ amplitude for a wire placed at 9.4σ of the beam barycenter versus multipole number

rather rapid. It however has no clear meaning. Actually, a naive calculation of the integrated b_{10} over the low-β quadrupoles arising either from the LR interactions or from the quadrupole field quality shows that the former is about 100 times stronger. This may give a clue at why the dynamics is strongly perturbed by the LR interactions and further motivates a correction method which deals with all multipoles.

2.6 Perturbation by the LR Magnetic Field

The perturbation of the amplitude of the betatron motion is given by:

$$\int_{l_{LR}} \sqrt{\beta_{x/y}}\, \frac{B_{y/x}}{B\rho}\, ds \tag{5}$$

If the β-functions are identical in the two planes, r_0, x and y scale with $\sqrt{\beta}$. Equations 2 and 3 show that the kick due to the $1/r$ magnetic lens becomes **independent** of β. This is in contrast with the usual multipolar lenses whose kicks scale like $\beta^{n/2}$.

In LHC, this result holds for all the LR encounters occurring in the experimental straight-section and a few in the low-β quadrupoles, i.e. about half of the encounters.

3 LR Correction Scheme

3.1 Principle of the Correction

We have shown that the kicks given by the strong beam can be lumped with good accuracy. The strong beam itself is well modelled by an electric current in a conductor. The perturbation to the beam does not depend on the external focusing, as long as it is the same in the two planes.

For the nominal beam separation in LHC, the multipole content of the LR magnetic field is still too rich to attempt a correction by standard magnetic multipoles. The obvious solution is to use a corrector made up of a wire running parallel to the weak beam trajectory whose electric current runs in the opposite direction of that of the strong beam.

The electric current integrated over the wire length should be equal to beam current integrated over the bunch length and the number of LR interactions, i.e.

$$27.36\text{A} \times \sqrt{2\pi} \times 7.7\text{cm} \times 15 \approx 80\text{A m} \tag{6}$$

3.2 Transverse Position of the Corrector

The perturbation of the amplitudes (Eq 5) shows that the corrector should be placed transversely at the same position as that of the strong beam, expressed in beam σ's (9.5σ in LHC v6). This provides an exact correction of the LR interactions occurring in the straight-section. We will investigate numerically the correction efficiency for the others occurring inside the low-β quadrupoles. At 9.5σ, the wire would be in the shadow of the secondary collimator but exposed to the tertiary halo. A development of this study will be to find the conditions under which the corrector can be further displaced by one or two σ.

3.3 Longitudinal Position of the Corrector

In the section common to the two rings, a solid LR corrector would of course intercept the strong beam. It is however possible to take advantage of the very large β-function in the low-β section (figure 6): the betatron phase hardly advances from the triplet to the D2 magnet. A corrector anywhere in this area is almost equivalent to a corrector at the LR interaction points. A second criterion is the 'locality' in betatron functions, as the latter change rapidly in this section. Since the majority of the LR interactions occur at positions where $\beta_x = \beta_y$,

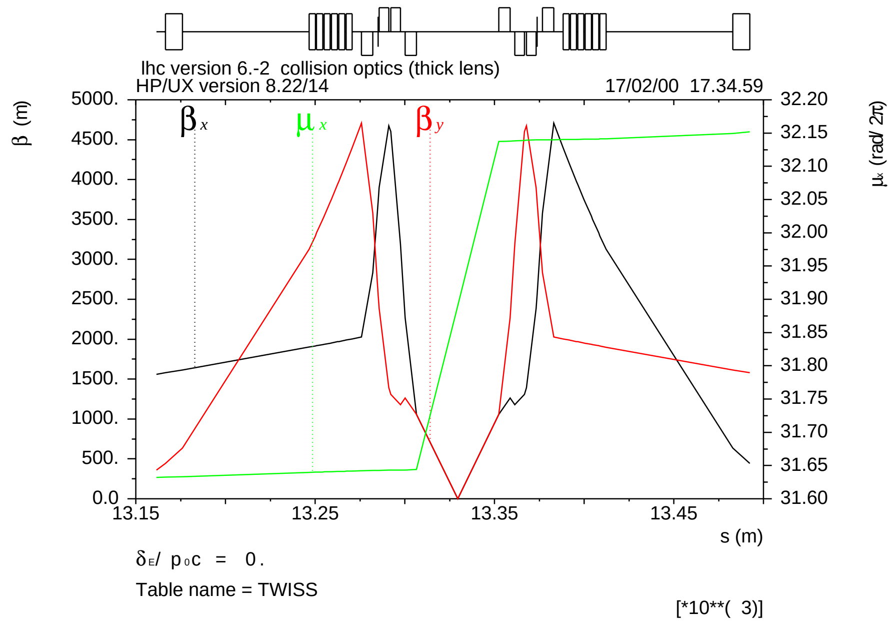


Figure 6: Twiss parameters in IR5, from D2.L5 to D2.R5

we choose to position the correctors at such a place. This is found at 112.322 m from IP5, about 41.1 m from the exit face of D2 towards D1. $\beta_x = \beta_y = 1935$ m with $\alpha_{x/y} \leq 18.5$ (figure 6). The average betatron phase shift from the LR interactions to the corrector is 2.6°.

To compensate correctly the LR forces, the corrector must be placed between the two beams or channels(figure 7), where the separation is about 100σ.

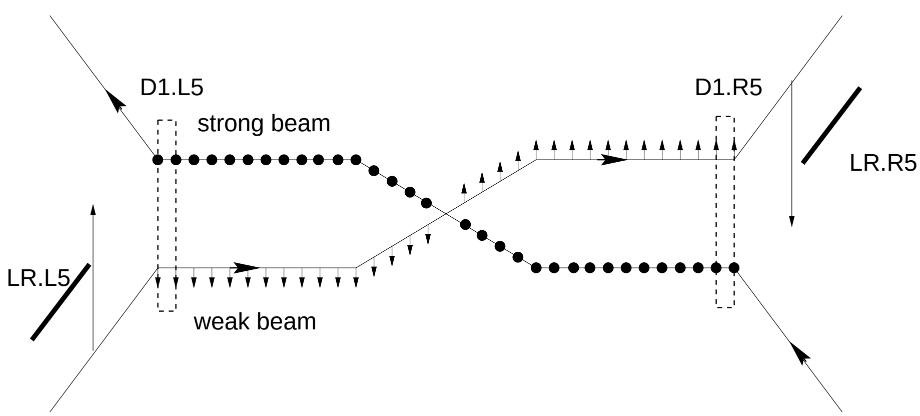


Figure 7: Position of the LR correctors

In this study, we assume two correctors per beam and per interaction points, i.e. one on each side of the IP's. This arrangement allows a correction of all perturbations, including the potential displacement of the closed orbit at the interaction point.

4 Numerical Results

4.1 Implementation of the $1/r$ lens in MAD

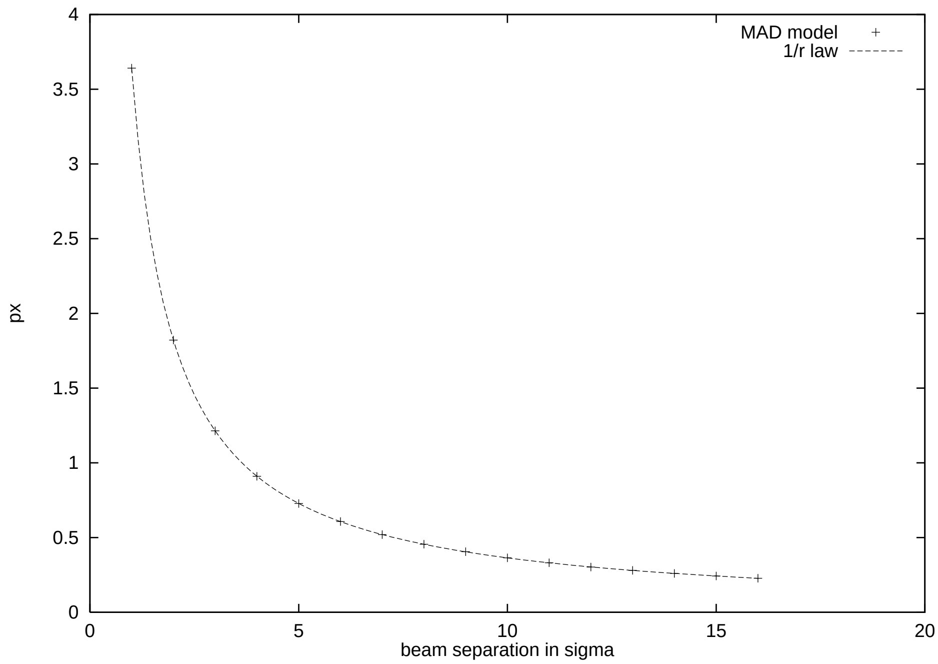


Figure 8: Comparison between the MAD model and an exact $1/r$ law

There is no magnetic lens implemented in MAD with a $1/r$ field. To model a LR corrector, we use instead a beam-beam lens whose beam size is artificially reduced ($\sigma_W/100$). The model is compared to an exact $1/r$ law on figure 8. The vertical scale is the kick p_x observed just after the beam-beam lens. The agreement is as expected.

4.2 Correction of the LR Interactions in the Experimental Straight-Section

This example is built artificially to verify the former analysis in a case where the correction should be exact. Only the LR interactions occurring in the experimental straight-section are retained (12 in total). They are transversely displaced by 10σ with respect to the orbit of the weak beam. Each LR interaction is allocated a charge of 15/6 to keep about constant the total perturbation (30 charges, one charge per interaction). The LR correctors are positioned on each side of the IP by matching at a position where the β-functions are the same in the two planes on the unperturbed machine, i.e. without the beam-beam effect. They are transversely displaced by 10σ and nominally powered at -15 charges each.

The criteria used to judge on the efficiency of the correction are: the betatron tunes, the horizontal closed orbit at the collision point and the largest extent of the tune footprint. The latter is calculated by tracking a set of initial conditions on circles in the x, y plane with radii ranging from $\sqrt{x^2+y^2} = 1\sigma$ to $\sqrt{x^2+y^2} = 6\sigma$. To allow a faster evaluation, the part of the lattice which does not include LR interactions is mapped at the order 3 in MAD.

The results are shown in Table 1. The correction is practically perfect. The closed orbit at the IP is hardly changed even before correction. This is due to the symmetry of the perturbation and is not true at an other azimuth.

Name	Q_x	Q_y	x_{IP5} μm	$\Delta Q(6\sigma)$ 10^{-3}
initial	.280000	.310000	0	.0033
12 LR's	.282020	.307991	.002	2.2
after correction	.280005	.309998	.003	.0092

Table 1: Correction of the twelve LR interactions in the straight-section

4.3 A Realistic Correction in IP5

In the following, all the 30 LR interactions are retained. The beam trajectories are now exactly simulated. The two correctors are positioned longitudinally in the same way as before. Their transverse displacement with respect to the weak beam orbit is 9.5σ, i.e. the average displacement of the strong beam at the LR interaction points. They are each powered at -15 beam charges (nominal correction). Table 2 shows that the compensation is still effective though not complete. Before correction, the beam is significantly displaced in IP1 ($.2\sigma$). The displacement becomes negligible after correction.

Name	Q_x	Q_y	x_{IP5} μm	x_{IP1} μm	$\Delta Q(6\sigma)$ 10^{-3}
bare machine	.280000	.310000	0	0	.0033
All LR's	.282422	.307553	.08	2.6	3.5
Nominal correction	.280200	.309766	.08	.09	.65
Optimized correction	.279904	.310063	.09	.25	.275

Table 2: Correction of the twelve LR interactions in the straight-section IR5

Figure 9 shows a footprint reduced by a factor of 5. If the current in the correctors is empirically increased by 13%, the footprint is divided by an order of magnitude as compared to the uncorrected tune spread. A similar result is obtained by slightly reducing the beam-corrector separation. In both cases, the reduction of the tune footprint is very significant. Following Eq. 4, a reduction of the tune footprint by a factor of 5 is equivalent to an increase of the crossing angle by at least 30%, assuming that the detuning is dominated by 4th and 6th order multipoles.

Since the betatron phase advance between perturbation and correction is small (2.6°), the tune footprint should be a rather faithful image of the other non-linear terms, e.g. the

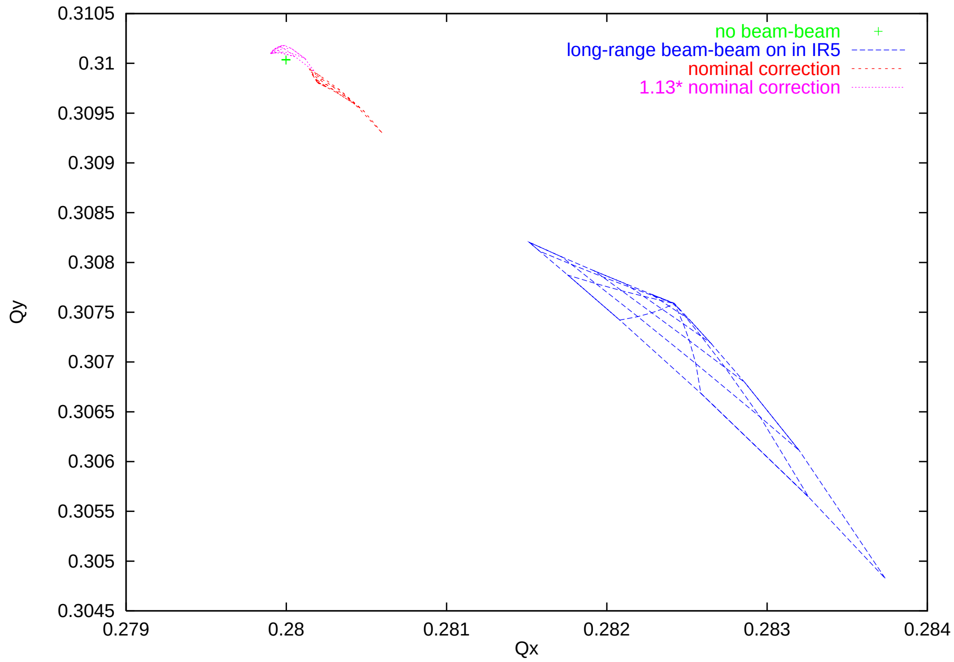


Figure 9: Minimization of the tune footprint

resonance driving terms. From $\sin(n \times 2.6°)$, the strength of resonances up to order 5 is reduced by a factor of at least 5. The strength of resonances up to order 11 is reduced by a factor of at least 2. With small adjustments of the currents in the two correctors, it is potentially possible to further minimize possibly offending resonances.

5 Conclusions

This note shows that the long-range beam-beam interactions, presently considered as the most drastic limitation of LHC performance, can be rather accurately corrected for both their linear and non-linear perturbations. The principle of the corrector is simple though departing from classical multipolar lenses. It requires a conductor running parallel to the beam and carrying a current of about 60 A over 2 m or 600 A over 20 cm. Ideally 8 such correctors would be needed, grouped in 4 boxes on either side of IP1 and IP5, placed at about 40 m from the exit face of D2 towards D1. If the corrector current can be pulsed at 1/15 of the bunch frequency, the whole spectrum of PACMAN effects can be corrected locally. An interesting extension would be to use a distribution of e.g. 3 conductors instead of one, allowing a larger physical transverse separation between the beam and the corrector for the same effective electro-magnetic separation. Before completing this study in finer details, it may seem interesting at this stage to consider whether and how such correctors can be implemented in practice. The requirement of pulsing the current does not seem a priori out of reach [12].

Acknowledgements

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